UCSB Department of Mathematics Course Outline MATH 8: Transition to Higher Mathematics

MATH 8 is a course that introduces students to the skill of writing mathematical proofs. To this end students will:

- Understand the logical structure of a mathematical statement with intent to develop a strategy to prove it,
- Understand and make use of definitions, axioms and previously proven statements,
- Use proper logic and mathematical convention to write rigorous arguments which prove statements in mathematics, and
- Write clearly using complete sentences to communicate their ideas.

The course is a prerequisite for most upper division mathematics courses. The sole purpose of MATH 8 is to ensure that students understand and can write mathematical proofs at a level necessary to succeed as mathematics majors. The emphasis is on developing strategies and communicating effectively in writing. In this regard assignments will have enough "non-template" questions so that students have to spend some time pondering solutions and then improve their writing through the rewriting process as initial ideas are reformulated more precisely.

Throughout the course students will use a variety of proof techniques. The following will be used by students in their writing:

- Direct proof,
- Proof using the contrapositive,
- Proof by contradiction,
- Proof by cases, and
- Inductive proof.

It should be noted that development of proof-writing is far more important than topic coverage. However the course also includes an introduction to several foundational topics in mathematics. The following topics are **essential**:

- Basic logic (conditional statements, logical connectives, quantifiers),
- Set theory (union, intersection, complement, power sets, Cartesian products),
- Number theory (divisibility of integers),

- Relations and functions (equivalence relations, injections, surjections, composition), and
- Cardinality and counting principles.

Instructors may include additional topics to promote interest in the material and to give students more opportunities to develop their proof-writing. Some **optional** topics include:

- Additional topics in set theory, for example ordinal numbers,
- Construction of the real or complex numbers,
- Modular arithmetic,
- Euclidean geometry,
- Elementary group theory,
- Combinatorics, or
- Computer-assisted proofs.

Instructors may choose from these or any other topics if time permits and the topics lend themselves to additional practice in proof-writing.

Recommended Texts:

- <u>A Concise Introduction to Pure Mathematics</u>, Martin Liebeck
- <u>A Transition to Advanced Mathematics</u>, Smith, Eggen & St. Andre
- How to Prove It: A Structured Approach, Daniel J. Velleman
- Introduction to Mathematical Structures and Proofs, Larry J. Gerstein
- <u>Proofs and Fundamentals: A First Course in Abstract Mathematics</u>, Ethan D. Bloch
- <u>Book of Proof</u>, Richard Hammack