2023-2024 GRADUATE COURSE DESCRIPTIONS

MATH 201 A-B-C (FWS), Craig/Labutin/Putinar, *Real Analysis*
Measure theory and integration. Point set topology. Principles of functional analysis. L^p spaces. The Riesz representation theorem. Topics in real and functional analysis.

MATH 206 A (F), Chandrasekaran, *Matrix Analysis & Computation*
Graduate level-matrix theory with introduction to matrix computations. SVDs, pseudoinverses, variational characterization of eigenvalues, perturbation theory, direct and interative methods for matrix computations.

MATH 206 B (W), Petzold, *Numerical Simulation*
Linear multistep methods and Runge-Kutta methods for ordinary differential equations: stability, order and convergence. Stiffness. Differential algebraic equations. Numerical solution of boundary value problems.

MATH 206 C (S), Ceniceros, *Numerical Solution of Partial Differential Equations - Finite Difference Methods*
Finite difference methods for hyperbolic, parabolic and elliptic PDEs, with application to problems in science and engineering. Convergence, consistency, order and stability of finite difference methods. Dissipation and dispersion. Finite volume methods. Software design and adaptivity.

MATH 206 D (F), Atzberger, *Numerical Solution of Partial Differential Equations - Finite Element Methods*
Weighted residual and finite element methods for the solution of hyperbolic, parabolic and elliptical partial differential equations, with application to problems in science and engineering. Error estimates. Standard and discontinuous Galerkin methods.

MATH 220 A-B-C (FWS), Liu/Morrison, *Modern Algebra*
Group theory, ring and module theory, field theory, Galois theory, other topics.

MATH 221 A (F), Cooper, *Foundations of Topology*
Metric spaces, topological spaces, continuity, Hausdorff condition, compactness, connectedness, product spaces, quotient spaces. Other topics as time allows.

MATH 221 B (W), Manin, *Homotopy Theory*
Homotopy groups, exact sequences, fiber spaces, covering spaces, van Kampen Theorem.

MATH 221 C (S), Bigelow, *Differential Topology*
Topological manifolds, differentiable manifolds, transversality, tangent bundles, Borsuk-Ulam theorem, orientation and intersection number, Lefschetz fixed point theorem, vector fields.

MATH 225 A (F), Castella, *Topics in Number Theory*
A first course in the arithmetic theory elliptic curves and modular forms. Topics to be covered include: Nagell{Lutz theorem, Mordell{Weil theorem, Hasse{Weil L-function, Birch and Swinnerton-Dyer conjecture, etc. in the first part; and Hecke operators, Eichler{Shimura isomorphism, modularity of elliptic curves, etc. in the second part.

MATH 225 B (W), Castella, *Topics in Analytic Number Theory*
We introduce some of the fundamental \modern" tools in the study of the arithmetic elliptic curves. A central theme will be Kolyvagin's results on the Birch and Swinnerton-Dyer conjecture, which we will complement with the necessary background: the theory of Complex Multiplication, Tate's local and global duality, etc. Towards the end, we will try to explain how these techniques extend to the study of more general Galois representations.

MATH 227 A (F), McCammond, *Advanced Topics in Topology*
Coxeter groups and Artin groups
Abstract: This course will be a general introduction to two large families of discrete groups. Coxeter groups are discrete groups generated by reflections and they can be described by very simple presentations. They include the symmetry groups of the Platonic solids and the symmetry groups of various Euclidean and hyperbolic tilings. Artin groups are the "braided" versions of Coxeter groups. Generally speaking, many deep results are known about all Coxeter groups, but very few results are known that apply to all Artin groups. We will be surveying both the known and the unknown.

MATH 227 B (W), Z. Wang, *Advanced Topics Topology*
2-tangles and (3+1)-TQFTs
Abstract: Knots, braids, and tangles in 3-space are parts of a ribbon category, which is the algebraic foundation of (2+1)-TQFTs. Similarly, 2-knots, 2-braids, and 2-tangles in 4-space should form some generalization of a ribbon category underlying (3+1)-TQFTs. But such a braided monoidal 2-category is much more complicated than the ribbon category of tangles. We will start with an introduction to 2-knots, 2-braids, and 2-tangles with an eye towards such a generalization with potential applications to (3+1)-TQFTs and topology of smooth 4-manifolds.

MATH 227 C (S), Manin, *Advanced Topics in Topology*
Nilpotent groups
Nilpotent groups can be defined algebraically: they are in some sense "almost abelian". Perhaps surprisingly, there is also a nice geometric characterization: a famous theorem of Gromov states that (finitely generated, virtually) nilpotent groups are exactly those with polynomial growth. In this course I will discuss nilpotent Lie groups and finitely generated nilpotent groups (scratch one of these and you'll see the other) and how their geometric and algebraic properties reflect each other.

Math 231A-B, (FW), Agboola, *Lie Groups and Lie Algebras*
Differentiable manifolds, definition and examples of Lie groups, Lie group-Lie algebra correspondence, nilpotent and solvable Lie algebras, classification of semi-simple Lie algebras over the complexes, representations of Lie groups and Lie algebras, special topics.

MATH 232 A (F), McCammond, *Algebraic Topology*
Singular homology and cohomology, exact sequences, Hurewicz theorem, Poincare duality.

Math 237A-B-C, (FWS), Morrison/X. Zhao, *Algebraic Geometry*
Affine/projective varieties, Hilbert's Nullstellensatz, morphisms of varieties, rational maps, dimension, singular/nonsingular points, blowing up of varieties, tangent spaces, divisors, differentials, Riemann{Roch theorem.

Special topics may include: elliptic curves, intersection numbers, Bezout's theorem, Max Noether's theorem.

MATH 240 A-B-C (FWS), Dai/Ye/Wei, ***Introduction to Differential Geometry and Riemannian Geometry***
Topics include geometry of surfaces, manifolds, differential forms, Lie groups, Riemannian manifolds, Levi-Civita connection and curvature, curvature and topology, Hodge theory. Additional topics such as bundles and characteristic classes, spin structures Dirac operator, comparison theorems in Riemannian geometry.

MATH 241 A (F), Ye, ***Advanced Topics in Differential Geometry***

MATH 241 B (W), Wei, ***Advanced Topics in Differential Geometry: Geometry and Topology of spaces with curvature bounds***

MATH 241 C (S), Dai, ***Advanced Topics in Differential Geometry: Laplace operator in geometry and mathematical physics***
The Laplace operator is everywhere, from the eigenvalue problems, to the curvature problems, to minimal surfaces, and to equations coming from mathematical physics. After introducing the preliminaries (the general philosophy, the Poisson equation on manifolds, elliptic operators and index theory perturbative theory of nonlinear equations), we will study three main examples: the Gaussian curvature equation on a surface, the Seiberg-Witten equation, and the Yamabe problem. The course will roughly follow a lecture notes by Simon Donaldson.

MATH 243 A-B-C (FWS), H. Zhou/Labutin/Harutyunyan, ***Ordinary Differential Equations***
Existence and stability of solutions, Floquet theory, Poincare-Bendixson theorem, invariant manifolds, existence and stability of periodic solutions, Bifurcation theory and normal forms, hyperbolic structure and chaos, Feigenbaum period-doubling cascade, Ruelle-Takens cascade.

MATH 260 AA (W), Birnir, ***Stochastic ODEs and PDEs, part 1***
Description: The basic existence theory of stochastic ODEs and PDEs using the book of Oksendal.

MATH 260 AA (S), Birnir, ***Nonlinear Stochastic PDEs, part 2***
Description: The existence theory of nonlinear stochastic PDEs and the associated statistical theory, with applications to turbulence, angiogenesis, earthquakes and geomorphology. Books by Da Prato and B. Birnir will be used as textbooks.

MATH 260EE (FWS), Cooper/Wei, ***Graduate Student Colloquium***
Topics in algebra, analysis, applied mathematics, combinatorial mathematics, functional analysis, geometry, statistics, topology, by means of lectures and informal conferences with members of faculty.

MATH 260H (W), S. Tang, ***High dimensional probability, approximation and statistical learning***
The course covers fundamental mathematical ideas for approximation and statistical learning problems in high dimensions. We will start with studying high-dimensional phenomena, through both the probability (concentration inequalities) and geometry (concentration phenomena). We then consider a variety of techniques and problems in high dimensional statistics, machine learning, and signal processing, ranging from dimensional reduction to classification and regression (with connections to approximation theory, Fourier analysis and wavelets, Reproducing kernel Hilbert spaces), to compressed sensing and matrix completion problems. We then consider graphs and networks, spectral graph theory, models of random graphs, and application to clustering. Finally we discuss problems at the intersection of statistical estimation, machine learning and dynamical system, in particular, the interacting particle system.

Reference list:
1. High dimensional probability: An introduction with applications in data science. R. Vershynin
2. High dimensional statistics. P. Rigollet
3. Ten lectures and Forty two open problems in the mathematics of data science
4. A distribution free theory of nonparametric regression
5. Lectures on spectral graph theory

MATH 260HH (F), Atzberger, *Topics in Machine Learning: Foundations and Applications*
This course covers mathematical topics relevant to machine learning to establish rigorous foundations and to provide guidelines for practical applications. As a rigorous basis for inference, the course will draw on results from functional analysis, optimization, convex analysis, statistical learning theory, and culminate in the discussion of active current topics. For example, the approximation abilities of deep learning with neural networks, formulations and training of unsupervised methods such as auto-encoders and GANs, and non-neural network approaches such as support vector machines, kernel methods, and probabilistic methods. A central emphasis will be on the development of rigorous mathematical theory and how this can be used to guide the design of machine learning algorithms, perform training, and carry out analysis to evaluate performance. The beginning introductory materials of the course will use the books "Foundations of Machine Learning," by M. Mohri, A. Rostamizadeh, and A. Talwalkar and "The Elements of Statistical Learning: Data Mining, Inference, and Prediction" by Hastie, Tibshirani, Friedman. The special topics part of the course will be based on materials developed by the instructor and from recent papers in the literature.

More details concerning specific topics can be found below. Sample of Topics:

- Introduction
  History, motivations, and recent developments.
  Statistical Learning Theory, PAC-Learnability, Bayesian Inference. Concentration Inequalities, High Dimensional Probability Theory. Sample Complexity Bounds , No-Free-Lunch Theorems.
  Survey of current topics in data sciences and applications.

- Topics in Supervised Learning
  Neural networks and deep learning approaches.
  Support vector machines, kernel methods, and probabilistic approaches. Parametric and non-parametric regression and sampling complexity. Decision trees, graphical models.

- Topics in Unsupervised Learning
  Manifold learning.
  Neural network auto-encoders and related feature extractors.
  Generative methods, Generative Adversarial Networks (GANs), and related approaches.

- Advanced Special Topics
  Non-linear optimization methods for machine learning, regularization, stochastic optimization.
  Theory of deep architectures (approximation theory / universality results). Reinforcement learning methods, stochastic approximation, and applications. Dimensionality reduction, sparse matrix methods.
  Emerging problems and approaches for applications in the sciences and engineering.

Bibliography:
1. Foundations of Machine Learning, by M. Mohri, A. Rostamizadeh, and A. Talwalkar.
2. The Elements of Statistical Learning Data Mining, Inference, and Prediction, T. Hastie, R. Tibshirani, J. Friedman, (2013).
The course also uses materials developed by the instructor and papers in the literature.

MATH 260J (F), Ceniceros, *Mathematical Aspects of High Dimensional Learning*

Description: This is a seminar style course on mathematical aspects of learning from data in high dimensional feature spaces. We will study shallow and deep networks from the approximation theory point of view and look at the problems of a) density, b) degree of approximation, and c) interpolation.

Some of the topics include the following:
1. Learning in reproducing kernel Hilbert spaces.
2. Approximation theory for the feedforward neural network model.
3. Clustering.
4. Dimensionality reduction.
5. Multiresolution for high dimensional data. Wavelets and treelets.
6. Convolutional neural networks.
7. Generative adversarial networks.
8. Some topics of continuous and discrete optimization for high dimensional learning.

Pre-requisites: Basic probability, linear algebra, analysis, and introductory numerical analysis.

Bibliography:
The course material will be drawn from research articles and from some textbooks relevant to each topic. Reading material will be provided for each topic.

MATH 260L (F), Garcia-Cervera, *Calculus of Variations*

In this course we will do an overview of the state of the art in some areas of the Calculus of Variations. We will illustrate the main concepts with applications of these variational methods to problems in Materials Science, Quantum Mechanics, PDEs, Differential Geometry, etc.

A typical example of the models we will study is given by the Ginzburg-Landau functional

$$F[u] = \frac{1}{2}\int_\Omega |\nabla u|^2\, dx + \frac{1}{4\varepsilon^2}\int_\Omega (1 - |u|^2)^2\, dx,$$

where $u : \Omega \to \mathbb{C}$ belongs to $H^1(\Omega;\mathbb{C})$, supplemented with certain types of boundary behavior. This functional models the behavior of superconductors, but is also related to the existence of Harmonic maps in two-dimensions, to the behavior of liquid crystals, and to the phase separation problem in complex fluids, such as a mixture of water and oil. In addition to the existence of minimizers, an important question we can ask ourselves is the behavior of the minimizers as $\varepsilon \to 0$. This last question often requires more advanced techniques, such as the notion of Γ-convergence, which is a type of asymptotic variational convergence.

The course should be accessible to students with a working knowledge of linear functional analysis (at the level of Math 201) and linear partial differential equations. Familiarity with the theory of Sobolev spaces is recommended, but we will do an introduction with the main aspects relevant for the course.

A brief tentative list of topics follows:
1. Theory of Distributions and Sobolev Spaces.
2. The Direct Method in the Calculus of Variations.
3. Weak Convergence, Lowe Semi-Continuity and Convexity,
4. Compensated Compactness and Concentration Compactness.
5. Variational Convergence: Homogenization and Γ-convergence.

References: Although some of the material will be extracted from published research articles, we will use some of the following references:
1. Introduction to the Calculus of Variations, by Bernard Dacorogna.
2. Variational Methods: Applications to Nonlinear Partial Differential Equations and Hamiltonian Systems, by Michael Struwe.
3. Convex Analysis and Variational Problems, by Ivar Ekeland and Roger Temam.
4. Weak ConvergenceMethods forNonlinearPartial Differential Equations, by Lawrence Evans.
5. Weak Continuity and Weak Lower Semicontinuity of Non-Linear Functionals, by Bernard Dacorogna.

Math 260L (W), Garcia-Cervera, *Operator Theory with Applications to Quantum Mechanics*
Most of the course will be dedicated to the spectral theory of bounded and unbounded operators in a Hilbert space. We will focus on concrete examples in the context of Quantum Mechanics. This course will also serve as an introduction to the study of the many-body Schrodinger Hamiltonian, and some of reduced models used in the study of quantum systems, such as Hartree-Fock, Thomas-Fermi, and Density-Functional Theory. We will also introduce some topics from the calculus of variations.

Here is a tentative list of topics:
1. Bounded operators: The Closed Graph theorem, resolvent set, and spectrum.
2. Unbounded operators on a Hilbert space: Self-adjointness and the Kato-Rellich condition.
3. Spectral theory of self-adjoint operators: Spectral measures and resolution of the identity.
4. Exponential decay of eigenfunctions.
5. Location of the essential spectrum and the RAGE theorem.
6. Perturbation theory of the discrete spectrum.
7. Periodic systems: the Bloch-Floquet transform.

To illustrate the main ideas, we will use the following model problems throughout the course:
1. One electron Hamiltonian and the Hydrogen atom.
2. Many-body Hamiltonian and the HVZ theorem.
3. Hartree-Fock, and Thomas-Fermi-von Weiszacker.

The course should be accessible to students with a working knowledge of linear functional analysis (at the level of Math 201), linear partial differential equations, and the theory of Sobolev spaces.

References: Although some of the material will be extracted from published research articles, we will use some of the following references:
1. Introduction to Spectral Theory With Applications to Schrodinger Operators, by P.D. Hislop and I.M. Sigal.

2. Mathematical Methods in Quantum Mechanics With Applications to Schrodinger Operators, by Gerald Teschl.
3. Functional Analysis Vol. I, by Michael Reed and Barry Simon.
4. Analysis of Operators Vol. IV, by Michael Reed and Barry Simon.
5. Perturbation theory for linear operators, by Tosio Kato.

Math 260Q (S), Goodearl, *Quantum Groups*
This course will offer an introduction to core ideas in a subject called, somewhat mysteriously, "Quantum Groups". It is a field which arose from mathematical physics in the 1980s and has since developed many connections with areas as diverse as representation theory, noncommutative geometry, and knot theory. Its historical origin was the study of the "quantum Yang-Baxter equation" in quantum statistical mechanics, solutions to which came from the representation theory of "quantized enveloping algebras" of Lie 3 algebras and led to "quantized algebras of functions" on Lie groups.

The prerequisite is just 220ABC or equivalent. Background in Lie theory, representation theory, or algebraic geometry is not assumed, but will be developed as needed, along with some relevant noncommutative algebra.

A sample list of topics to be discussed follows:
{ Affine algebraic varieties, polynomial function algebras, and their quantizations
{ Lie algebras, enveloping algebras, and their quantizations
{ The quantum Yang-Baxter equation and R-matrices
{ Hopf algebras
{ Hopf duality between (quantized) algebras of functions on Lie groups and (quantized) enveloping algebras of their Lie algebras

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MATH 501 (F), Garfield, *Teaching Assistant Training*
Consideration of ideas about the process of learning mathematics and discussion of approaches to teaching.