2022-2023 GRADUATE COURSE DESCRIPTIONS

MATH 201 A-B-C (FWS), Harutyunyan/H. Zhou, ***Real Analysis***
Measure theory and integration. Point set topology. Principles of functional analysis. L^p spaces. The Riesz representation theorem. Topics in real and functional analysis.

MATH 202 A-B-C (FWS), Labutin/Putinar, ***Complex Analysis***
Analytic functions. Complex integration. Cauchy's theorem. Series and product developments. Entire functions. Conformal mappings. Topics in complex analysis.

MATH 206 A (F), Chandrasekaran, ***Matrix Analysis & Computation***
Graduate level-matrix theory with introduction to matrix computations. SVDs, pseudoinverses, variational characterization of eigenvalues, perturbation theory, direct and interative methods for matrix computations.

MATH 206 B (W), Petzold, ***Numerical Simulation***
Linear multistep methods and Runge-Kutta methods for ordinary differential equations: stability, order and convergence. Stiffness. Differential algebraic equations. Numerical solution of boundary value problems.

MATH 206 C (S), Ceniceros, ***Numerical Solution of Partial Differential Equations - Finite Difference Methods***
Finite difference methods for hyperbolic, parabolic and elliptic PDEs, with application to problems in science and engineering. Convergence, consistency, order and stability of finite difference methods. Dissipation and dispersion. Finite volume methods. Software design and adaptivity.

MATH 206 D (F), Garcia-Cervera, ***Numerical Solution of Partial Differential Equations - Finite Element Methods***
Weighted residual and finite element methods for the solution of hyperbolic, parabolic and elliptical partial differential equations, with application to problems in science and engineering. Error estimates. Standard and discontinuous Galerkin methods.

MATH 220 A-B-C (FWS), Morrison/Castella/Goodearl, ***Modern Algebra***
Group theory, ring and module theory, field theory, Galois theory, other topics.

MATH 221 A (F), Manin, ***Foundations of Topology***
Metric spaces, topological spaces, continuity, Hausdorff condition, compactness, connectedness, product spaces, quotient spaces. Other topics as time allows.

MATH 221 B (W), Cooper, ***Homotopy Theory***
Homotopy groups, exact sequences, fiber spaces, covering spaces, van Kampen Theorem.

MATH 221 C (S), Cooper, ***Differential Topology***
Topological manifolds, differentiable manifolds, transversality, tangent bundles, Borsuk-Ulam theorem, orientation and intersection number, Lefschetz fixed point theorem, vector fields.

MATH 225 A (F), Z. Liu, *Class Field Theory*

Class Field Theory is widely regarded as one of the crowning achievements of the early twentieth century mathematics, and its extension to the non-abelian setting underlies much of the current research work in algebraic number theory.

In this course, we will first state the main theorems in global class field theory which relate (generalized) ideal class groups of a number field to its abelian extensions. Then we will introduce local fields and adeles, state the main theorems of local class field theory, and introduce the adelic formulation of class field theory. Then we will discuss Galois cohomology and proofs of the main theorems.

Math 225 B (W), Y. Zhang, *Topics in Analytic Number Theory*

MATH 225 C (S) – X. Zhao*, O-minimal theory and applications*

Tame topology results have in the last decade found a number of important applications to algebraic and arithmetic geometry. The idea is to rebuild the whole mathematics in a world only allowing certain types of functions, like polynomials and logarithmic functions. The key ingredient is the use of o-minimality, a theory from mathematical logic, to pass between the geometry of an algebraic variety and that of its uniformizing space. This machinery is actively used recently in number theory (Shimura varieties and unlikely intersection), algebraic geometry (arithmetic of Hodge theory) and real analytic geometry (various definability questions).

The goal of this course is to give a tour through the main applications of o-minimal theory towards arithmetic questions in Hodge theory. We start by introducing the basic notions of o-minimal geometry with a view towards the two algebraization theorems of Pila–Wilkie and Peterzil–Starchenko. We then show how these results are applied to prove the Ax-Schanuel conjecture and the algebraicity of Hodge loci.

No prerequisite on logic or Hodge theory is required. We will review the relevant background during the course.

MATH 227 A (F), Cooper, *Advanced Topics in Topology*
This course will cover various topics in low dimensional topology and geometric structures on manifolds including some of the following:

Hyperbolic geometry, Teichmuller Space, measured foliations, Thurston compactification, classification of diffeomorphisms of surfaces

3-Manifolds, geometrization theorem, knot theory.

Projective geometry, convex projective structures, geometric transitions.

Higher Teichmuller theory, Fock-Goncharov coordinates, mixed structures, a Thurston-type compactification.

I will point out some of the many intriguing open questions in these areas.

MATH 227 B (W), Z. Wang, *Advanced Topics in Geometric and Algebraic Topology*
Mixed-state TQFTs and applications

Abstract: A generalization of Atiyah type TQFTs from pure states to mixed states in the sense that the Hilbert space of pure states associated to a space manifold is replaced by a quantum coherent space related to density matrices is proposed in arxiv 2110.13946. Atiyah type TQFT is a symmetric monoidal functor from the Bord category of manifolds to the category Vec of finite dimensional vector spaces, while in mixed-state TQFTs the target category Vec is replaced by QCS--the category of quantum coherent spaces, so a mixed-state TQFT is simply a symmetric monoidal functor from Bord to QCS. We also discuss how to construct interesting examples and their applications in topology, physics, and quantum computing.

MATH 227 C (S), McCammond, *Advanced Topics in Geometric and Algebraic Topology*
Reflections, Braids and Polynomials

Abstract: In this course I will cover the basic foundations of groups generated by reflections (Coxeter groups) and their braided complexified cousins (Artin groups). Towards the end of the course, I will discuss new research that shows how properties of classical polynomials in one complex variable can be used to investigate the structure of a classifying space for the braid group.

MATH 232 A-B (FW), Manin/Cooper, *Algebraic Topology*
Singular homology and cohomology, exact sequences, Hurewicz theorem, Poincare duality.

MATH 236 A (W), H. Zimmermann, *Homological Algebra*

We will start by assembling a minimal kit of concepts and results from category theory. Exactness problems for Hom and tensor functors will lead us to the study of projective, injective, and flat objects in abelian categories; these objects provide the cornerstones for subsequent homological constructions.

Among the main aims is a clean introduction of derived functors, with Ext and Tor serving as the primary examples. The Ext-functor, in particular, will be considered from various angles. We will follow with applications, such as (homological) dimension theory. Moreover, we will supplement the development of derived functors with structure results for Ext- and Tor- groups which are relevant to applications.

MATH 236 B (S), X. Zhao, *Homological Algebra*

In the second quarter of the course, we will introduce a categorical approach to work with complexes and cohomology. For an abelian category (e.g. category of modules over a ring, category of sheaves of abelian groups over a topological space, etc), we construct its derived category. Roughly speaking, the derived category consists of complexes of objects in the abelian category, and two complexes having the same cohomology are now considered isomorphic to each other. The general framework of triangulated categories and localizations will be introduced to construct such a category and to study its properties.

The second part of the course introduces derived functor. After showing the construction, we will focus on derived functors most used in algebra and algebraic geometry, and use them to construct and compute several invariants (Hochschild and cyclic homology, higher direct image sheaves, etc). An introduction to spectral sequences with applications towards computing the composition of derived functors will be covered.

Prerequisite: Math 220ABC and 236A, or consent of instructor. The students should have a solid background in module theory. We will use sheaves on topological spaces as one major example, the basic definitions will be covered in class.

MATH 240 A-B-C (FWS), Dai/Ye/Wei, *Introduction to Differential Geometry and Riemannian Geometry*

Topics include geometry of surfaces, manifolds, differential forms, Lie groups, Riemannian manifolds, Levi-Civita connection and curvature, curvature and topology, Hodge theory. Additional topics such as bundles and characteristic classes, spin structures Dirac operator, comparison theorems in Riemannian geometry.

MATH 241 A (F), Ye, *Topics in Differential Geometry*

MATH 241 B (W), Wei, *Topics in Differential Geometry*

Integral Ricci curvature lower bound is much weaker than pointwise bound. Many geometric problems lead to integral curvatures; for example, the isospectral problems, geometric variational problems and extremal metrics, and Chern-Weil's formula for characteristic numbers. Thus, integral curvature bounds can be viewed as an optimal curvature assumption here. We will study the geometry and topology of manifolds with integral curvature bounds.

MATH 241 C (S), Dai, *Topics in Differential Geometry*

Laplace operator in geometry and mathematical physics

The Laplace operator is everywhere, from the eigenvalue problems, to the curvature problems, to minimal surfaces, and to equations coming from mathematical physics. After introducing the preliminaries (the general philosophy, the Poisson equation on manifolds, elliptic operators and index theory perturbative theory of nonlinear equations), we will study three main examples: the Gaussian curvature equation on a surface, the Seiberg-Witten equation, and the Yamabe problem. The course will roughly follow a lecture notes by Simon Donaldson.

MATH 246 A-B-C (FWS) H. Zhou/Harutyunyan, *Partial Differential Equations*

Existence and stability of solutions, Floquet theory, Poincare-Bendixson theorem, invariant manifolds, existence and stability of periodic solutions, Bifurcation theory and normal forms, hyperbolic structure and chaos, Feigenbaum period-doubling cascade, Ruelle-Takens cascade.

MATH 260 AA (W), Hu, *High-dimensional Control, Games, Learning Theory and Algorithm*

Many scenarios in finance, economics, management science, and engineering can be formuated as high-dimensional stochastic control and game problems. Stochastic control problems study the agent's rational behavior with the existence of uncertainty in observations or in the noise that drives the evolution of the system. Stochastic differential games, as an offspring of game theory and stochastic control, provide the modeling and analysis of interactive agents' conflict in the context of a dynamical system with uncertainty. Analytical solutions usually only exist under simple models, and one needs to resort to numerical algorithms beyond them. However, when the system is of high-dimension, conventional numerical methods soon lose their efficiency. This topic course will systematically introduce the theory and algorithms of deep learning and reinforcement learning to address the high-dimensional (and infinite- dimensional) stochastic control and game problems, with discussions on the applications to finance and economics.

MATH 260EE (FWS) Cooper, ***Graduate Student Colloquium***
Topics in algebra, analysis, applied mathematics, combinatorial mathematics, functional analysis, geometry, statistics, topology, by means of lectures and informal conferences with members of faculty.

MATH 260H (S) S. Tang, ***High dimensional probability, approximation and statistical learning***

The course covers fundamental mathematical ideas for approximation and statistical learning problems in high dimensions. We will start with studying high-dimensional phenomena, through both the probability (concentration inequalities) and geometry (concentration phenomena). We then consider a variety of techniques and problems in high dimensional statistics, machine learning, and signal processing, ranging from dimensional reduction to classification and regression(with connections to approximation theory, Fourier analysis and wavelets, Reproducing kernel Hilbert spaces), to compressed sensing and matrix completion problems. We then consider graphs and networks, spectral graph theory, models of random graphs, and application to clustering. Finally we discuss problems at the intersection of statistical estimation, machine learning and dynamical system, in particular, the interacting particle system.

Reference list:

1. High dimensional probability: An introduction with applications in data science. R. Vershynin

2. High dimensional statistics. P. Rigollet
3. Ten lectures and Forty two open problems in the mathematics of data science

4. A distribution free theory of nonparametric regression
5. Lectures on spectral graph theory

MATH 260HH (F), Atzberger, ***Topics in Machine Learning: Foundations and Applications***

This course covers mathematical topics relevant to machine learning to establish rigorous foundations and to provide guidelines for practical applications. As a rigorous basis for infer- ence, the course will draw on results from functional analysis, optimization, convex analysis, statistical learning theory, and culminate in the discussion of active current topics. For ex- ample, the approximation abilities of deep learning with neural networks, formulations and training of unsupervised methods such as auto-encoders and GANs, and non-neural network approaches such as support vector machines, kernel methods, and probabilistic methods. A central emphasis will be on the development of rigorous mathematical theory and how this can be used to guide the design of machine learning algorithms, perform training, and carry out analysis to evaluate performance. The beginning introductory materials of the course will use the books "Foundations of Machine Learning," by M. Mohri, A. Rostamizadeh, and A. Talwalkar and "The Elements of Statistical Learning: Data Mining, Inference, and Pre- diction" by Hastie, Tibshirani, Friedman. The special topics part of the course will be based on materials developed by the instructor and from recent papers in the literature.

More details concerning specific topics can be found below. Sample of Topics:

- Introduction
  History, motivations, and recent developments.
  Statistical Learning Theory, PAC-Learnability, Bayesian Inference. Concentration Inequalities, High Dimensional Probability Theory. Sample Complexity Bounds , No-Free-Lunch Theorems.
  Survey of current topics in data sciences and applications.

- Topics in Supervised Learning
  Neural networks and deep learning approaches.
  Support vector machines, kernel methods, and probabilistic approaches. Parametric and non-parametric regression and sampling complexity. Decision trees, graphical models.

- Topics in Unsupervised Learning
  Manifold learning.
  Neural network auto-encoders and related feature extractors.
  Generative methods, Generative Adversarial Networks (GANs), and related approaches.

- Advanced Special Topics

  Non-linear optimization methods for machine learning, regularization, stochastic optimization.

  Theory of deep architectures (approximation theory / universality results). Reinforcement learning methods, stochastic approximation, and applications. Dimensionality reduction, sparse matrix methods.
  Emerging problems and approaches for applications in the sciences and engineering.

Bibliography:

1. Foundations of Machine Learning, by M. Mohri, A. Rostamizadeh, and A. Talwalkar.

2. The Elements of Statistical Learning Data Mining, Inference, and Prediction, T. Hastie, R. Tibshirani, J. Friedman, (2013).

The course also uses materials developed by the instructor and papers in the literature.

MATH 260J (S), H. Zhou, *Introduction to X-ray and Radon transforms*

In inverse problems one attempts to determine the interior properties of a medium by applying various non-intrusive methods. In this topics course, we will introduce the mathematical analysis and practical inversion of the X-ray and Radon transforms, which are concerned with recovering a function or tensor from its integrals along straight lines or planes. It is the theoretical underpinning for several medical imaging methods, such as Computed Tomography (CT) and Positron Emission Tomography (PET). If time permits, we will also give a brief introduction to their generalizations in non-trivial geometry and connections to other inverse problems. There are no prerequisites, but basic knowledge of PDEs and Fourier analysis will be helpful for taking the course.

References:
Frank Natterer, The Mathematics of Computerized Tomography

Math 260L, (F), Harutyunyan, *Introduction to composite materials*

A composite material is a combination of two or more materials with different physical and chemical properties. Composites are abundant in nature. New composites are designed and created by engineers to do certain jobs that the materials in hand can't do. The physical and chemical properties of a composite depend on the physical and chemical properties of the composing parts and the composition geometry. For periodic geometries the problem studying the properties of a composite is more approachable. In this introduction course we will present the basics of the theory of composites that study physical (mechanical, electrical, etc.) properties of composites materials, that are made of two materials, and that have simple geometric structures. Original examples such as checkerboards, spherical inclusions, layered materials (laminates), etc. will be presented. A basic theory of ”Homogenization” will be presented as well.

MATH 260L, (W), Shagholian, ***TBA***

MATH 260Q, (W), Castella, ***Introduction to Iwasawa theory***

Iwasawa theory is the study of objects of arithmetic interest as they vary in p-adic families. It was originally introduced by Iwasawa in the context of class groups of cyclotomic fields, and transferred to the setting of elliptic curves by Mazur in the 1970s. This course will be an introduction to the basic methods and results of Iwasawa theory, with a special emphasis on the context of elliptic curves and its application to the Birch–Swinnerton-Dyer conjecture.

MATH 501 (F), Garfield, ***Teaching Assistant Training***

Consideration of ideas about the process of learning mathematics and discussion of approaches to teaching.