2021-2022 GRADUATE COURSE DESCRIPTIONS

MATH 201 A-B-C (FWS), Harutyunyan/H. Zhou, *Real Analysis*
Measure theory and integration. Point set topology. Principles of functional analysis. L^p spaces. The Riesz representation theorem. Topics in real and functional analysis.

MATH 206 A (F), Chandrasekaran, *Matrix Analysis & Computation*
Graduate level-matrix theory with introduction to matrix computations. SVDs, pseudoinverses, variational characterization of eigenvalues, perturbation theory, direct and interative methods for matrix computations.

MATH 206 B (W), Petzold, *Numerical Simulation*
Linear multistep methods and Runge-Kutta methods for ordinary differential equations: stability, order and convergence. Stiffness. Differential algebraic equations. Numerical solution of boundary value problems.

MATH 206 C (S), Ceniceros, *Numerical Solution of Partial Differential Equations - Finite Difference Methods*
Finite difference methods for hyperbolic, parabolic and elliptic PDEs, with application to problems in science and engineering. Convergence, consistency, order and stability of finite difference methods. Dissipation and dispersion. Finite volume methods. Software design and adaptivity.

MATH 206 D (F), Garcia-Cervera*, Numerical Solution of Partial Differential Equations - Finite Element Methods*
Weighted residual and finite element methods for the solution of hyperbolic, parabolic and elliptical partial differential equations, with application to problems in science and engineering. Error estimates. Standard and discontinuous Galerkin methods.

MATH 220 A-B-C (FWS), Morrison/Castella/Goodearl, *Modern Algebra*
Group theory, ring and module theory, field theory, Galois theory, other topics.

MATH 221 A (F), Manin, *Foundations of Topology*
Metric spaces, topological spaces, continuity, Hausdorff condition, compactness, connectedness, product spaces, quotient spaces. Other topics as time allows.

MATH 221 B (W), Cooper, *Homotopy Theory*
Homotopy groups, exact sequences, fiber spaces, covering spaces, van Kampen Theorem.

MATH 221 C (S), Cooper, *Differential Topology*
Topological manifolds, differentiable manifolds, transversality, tangent bundles, Borsuk-Ulam theorem, orientation and intersection number, Lefschetz fixed point theorem, vector fields.

MATH 225 A (F), Agboola, *Algebraic Number Theory*
This course is intended to be a quarter-long introduction to basic algebraic number theory. It would be helpful (but not essential) to take this course if you are planning to take 225BC in the Winter and Spring. A list of topics that will be covered includes:

Basic commutative algebra: Noetherian properties, integrality, rings of integers.

More commutative algebra: Dedekind domains, unique factorisation of ideals, localisation.

Norms, traces and discriminants.

Decomposition of prime ideals in an extension field.

Class numbers and units. Finiteness of the class number: Minkowski bounds. Dirichlet's unit theorem. Explicit calculation of units.

Decomposition of prime ideals revisited: the decomposition group and the inertia group associated to a prime ideal. A nice proof of quadratic reciprocity.

The prerequisites for this course are a solid knowledge of the basic first-year graduate courses in algebra and analysis, and a level of mathematical maturity appropriate for an advanced graduate course.

Some references:

"Algebraic Theory of Numbers", by P. Samuel (recently reprinted as a Dover paperback)
"Algebraic Number Theory" by A. Frohlich and M. J. Taylor (CUP).
"Algebraic Number Theory", by S. Lang (Springer).
"Algebraic Number Theory", by J. Neukirch (Springer)

Math 225 BC (WS) – Z. Liu, *Introduction to Modular Forms*
Course description: A modular form is a holomorphic function on the complex upper- half plane invariant under the action of a congruence subgroup of $SL_2(Z)$ and satisfying a growth condition. The foundation for the modern theory of modular forms was created by Hecke in the 1920's and 1930's. Two major developments of the theory began to unfold around 1970. One is its applications in the study of the arithmetics of elliptic curves and Selmer groups of Galois representations. The other development was the beginning of the Langlands program seeking to establish a correspondence between Galois representations and automorphic representations of reductive groups.
The purpose of this course is to explore some of the basic ideas in the theory of modular forms and their connection with elliptic curves and automorphic representations.

Topics to be covered: This is planned as a two-quarter (winter and spring) course.

Topics in the first quarter include: definition of modular forms, Eisenstein series, Hecke operators, modular curves, the L-function of modular forms, the Eichler– Shimura isomorphism, the Eichler–Shimura relation, the statement of the modularity lifting theorems.

Topics in the second quarter include: the adeles, Tate's thesis, the adelization of classical modular forms, the Rankin–Selberg method, Whittaker models, Maass–Shimura differential operators.

Prerequisites: Math 220 ABC

Grading: homework 50%, take-home final exam 50%

References:
F. Diamond and J. Shurman, A first course in modular forms, Graduate Texts in Mathematics, 228. Springer-Verlag, New York, 2005.

H. Hida, Elementary theory of L-functions and Eisenstein series, London Mathematical Society Student Texts, 26. Cambridge University Press, Cambridge, 1993.

MATH 227 B (W), Z. Wang, *Advanced Topics in Geometric and Algebraic Topology*
Topology of configuration spaces and applications
Abstract: The tangent bundle of a smooth manifold is an important invariant of the manifold, but in low dimensional topology, tangent bundle is close to be a homotopy invariant. Quantum topology highlights an old family of non-homotopy invariant of a space---the sequence of configuration spaces. It is obviously not a homotopy invariant of the space as illustrated by a point and the disk. We will cover the basic theory of configuration spaces, and applications from Jones polynomial to anyon physics if time permits.

MATH 227C (S), Manin, *Advanced Topics in Geometric and Algebraic Topology*
Geometry in the large
Abstract: This course will focus on properties of metric spaces that are invariant up to quasi-isometry. Informally, this means that we have to look at the space from far away, so that we can't, for example, distinguish the Euclidean plane from just its integer points. Many, but not all, of the spaces will be Cayley graphs of groups. In particular, we will show that there are spaces that look a heck of a lot like Cayley graphs, but aren't.

MATH 232 A-B (WS), Long/Bigelow, *Algebraic Topology*
Singular homology and cohomology, exact sequences, Hurewicz theorem, Poincare duality.

MATH 231 A-B (FW), Z. Liu/Goodearl
Differentiable manifolds, definition and examples of lie groups, lie group-lie algebra correspondence, nilpotent and solvable lie algebras, classification of semi-simple lie algebras over the complexes, representations of lie groups and lie algebras, special topics.

MATH 237 A-B (FW), Morrison/X. Zhao
Affine/projective varieties, Hilbert's Nullstellensatz, morphisms of varieties, rational maps, dimension, singular/nonsingular points, blowing up of varieties, tangent spaces, divisors, differentials, Riemann-Roch theorem. Special topics may include: elliptic curves, intersection numbers, Bezout's theorem, Max Noether's theorem.

MATH 237 C (S), H. Zimmermann, *Algebraic groups and applications to representation theory*
This course will be the third quarter of the 237 sequence on algebraic geometry next year.

We will develop the basics of linear algebraic groups (i.e., groups which carry a structure of affine algebraic variety so that the group operations are morphisms of varieties). Then we will discuss actions of such groups on other (not necessarily affine) varieties.

Subsequently, we will apply this theory to the understanding of representations of finite dimensional algebras. One of our goals in this connection will be to prove a central theorem characterizing the oriented graphs that give rise to path algebras of finite representation type.

MATH 240 A-B-C (FWS), Dai/Wei/Ye, *Introduction to Differential Geometry and Riemannian Geometry*
Topics include geometry of surfaces, manifolds, differential forms, Lie groups, Riemannian manifolds, Levi-Civita connection and curvature, curvature and topology, Hodge theory. Additional topics such as

bundles and characteristic classes, spin structures Dirac operator, comparison theorems in Riemannian geometry.

MATH 241 A (F), Wei, *Topics in Differential Geometry*
Integral curvature bound is much weaker than pointwise bound. Many geometric problems lead to integral curvatures; for example, the isospectral problems, geometric variational problems and extremal metrics, and Chern- Weil's formula for characteristic numbers. Thus, integral curvature bounds can be viewed as an optimal curvature assumption here. We will study the geometry and topology of manifolds with integral Ricci curvature lower bound by developing comparison theory for integral curvature, and manifolds with small $L^{n/2}$ bound on curvature tensor by using Ricci flow smoothing.

MATH 241 B (W), Ye, *Topics in Differential Geometry*
This course is an introduction to Einstein manifolds and Ricci flow. The basic theory of Einstein manifolds and Ricci flow will be presented.

MATH 241 C (S), Dai, *Topics in Differential Geometry*
Witten Deformation and Geometry of Landau-Ginzburg Models

Witten deformation is a deformation of the de Rham complex introduced in an extremely influential paper by Witten. Witten deformation on closed manifolds has found many beautiful applications, from the analytic proof of Morse inequalities to the development of Floer homology theory. Recent development in mirror symmetry, in particular the Calabi-Yau/Landau-Ginzburg correspondence has highlighted the importance of mathematical study of Landau-Ginzburg models. This leads us to a whole range of questions on the Witten deformation on non-compact manifolds. The course will introduce this circle of ideas surrounding Witten deformation and the geometry of Landau-Ginzburg models.

MATH 243 A-B-C (FWS) Atzberger/Labutin/Birnir, *Ordinary Differential Equations*
Existence and stability of solutions, Floquet theory, Poincare-Bendixson theorem, invariant manifolds, existence and stability of periodic solutions, Bifurcation theory and normal forms, hyperbolic structure and chaos, Feigenbaum period-doubling cascade, Ruelle-Takens cascade.

MATH 260 AA (W) Garcia-Cervera, *Mathematical Foundations of Electronic Structure Theory*
Description of the course:
A complete quantum mechanical description of an electronic system requires the solution of the many-body Schrodinger equation. The numerical approximation of this equation, however, is impractical: using a straightforward numerical discretization, the number of degrees of freedom grows exponentially with the number of electrons, and therefore only systems with very few atoms can be considered. As a result, a number of reduced models have appeared in the literature, and are intensely use in Physics, Chemistry, and Materials Science for the study and development of new materials with desirable properties. These problems have been the source of a significant amount
of work in the Mathematical community. This course is an introduction to the study of the Schr¨odinger Hamiltonian, and some of reduced models used in the study of quantum systems, such as Hartree-Fock, Thomas-Fermi, and Kohn-Sham Density-Functional Theory. To do this, we will introduce some topics from operator theory and the calculus of variations. However, the course will be partly conducted as a research seminar, with discussions of current areas of research and open problems.

Tentative list of topics:
1. One Electron Hamiltonians: The Hydrogen Atom.
2. Many-Body Schr¨odinger Hamiltonian: The HVZ Theorem.
3. Periodic Systems and the study of solids: Bloch theory.

4. Perturbation Theory.
5. Exponential decay of eigenfunctions: Localization.
6. Hartree-Fock, Thomas-Fermi, and Density-Functional Theory (DFT).
7. Beyond DFT: Green's functions and Many-Body methods.

Prerequisites: Basic knowledge of Functional Analysis and PDEs (at the level of Math 201 and Math 243).

References: Although some of the material will be extracted from published research articles, we will use some of the following references:
1. Functional Analysis, by Michael Reed and Barry Simon.
2. Analysis of Operators, by Michael Reed and Barry Simon.
3. Quantum Mechanics, by Messiah.
4. Peturbation Theory of Linear Operators, by Tosio Kato.
5. Quantum Theory of Many-Particle Systems, by Alexander L. Fetter and John Dirk Walecka.

MATH 260EE (FWS) Wei/Cooper, ***Graduate Student Colloquium***
Topics in algebra, analysis, applied mathematics, combinatorial mathematics, functional analysis, geometry, statistics, topology, by means of lectures and informal conferences with members of faculty.

MATH 260H (W) S. Tang, ***Mathematical foundation of statistical and machine learning***
Description: The course covers fundamental mathematical ideas for approximation and statistical learning problems in high dimensions. We will start with studying high-dimensional phenomena, through both the probability (concentration inequalities) and geometry (concentration phenomena). We then consider a variety of techniques and problems in high dimensional statistics, machine learning, and signal processing, ranging from dimensional reduction to classification and regression(with connections to approximation theory, Fourier analysis and wavelets, Reproducing kernel Hilbert spaces), to compressed sensing and matrix completion problems. We then consider graphs and networks, spectral graph theory, models of random graphs, and application to clustering. Finally we discuss problems at the intersection of statistical estimation, machine learning and dynamical system, in particular, the interacting particle system. Prerequisites: Linear algebra (MATH108 ABC), real and functional analysis (Math124B, 201A-C), and probability and statistics (PSTAT120A), or permission of instructor.

Reference list:
High dimensional probability: An introduction with applications in data science. R. Vershynin
High dimensional statistics. P. Rigollet
Ten lectures and Forty two open problems in the mathematics of data science
A distribution free theory of nonparametric regression
Lectures on spectral graph theory

MATH 260HH (F) R. Hu, ***Deep Learning for Stochastic Control and Games***
Stochastic control and games describe the behavior of a population of interactive agents among which everyone makes his/her optimal decision in a common environment. Many scenarios in finance, economics, management science, and engineering can be formulated as stochastic control and game problems. Differential games, as an offspring of game theory and optimal control, provide the modeling and analysis of conflict in the context of a dynamical system. Computing Nash equilibria is one of the core objectives in differential games, with a major bottleneck coming from the notorious curse of dimensionality.

In this topic course, we will systematically introduce the theory and algorithms of deep learning to address the high-dimensional (and infinite-dimensional) stochastic control and game problems, with discussions on the applications to finance and economics.

Tentative list of topics:
1. Intro to neural networks, feedforward, convolutional, recurrent NN, the universal approximation theorem.
2. Single agent problems: DGM method and Deep BSDE.
3. Finite agents' stochastic games: deep fictitious play method.
4. Control and games of mean-field type: fictitious play-based algorithm and FBSDE approach.
5. Model-free problems via reinforcement learning.

Main references are recent publications, including but not limited to:
1. DGM: A deep learning algorithm for solving partial differential equations, by Justin Sirignano and Konstantinos Spiliopoulos,
2. Deep learning-based numerical methods for high-dimensional parabolic partial differential equations and backward stochastic differential equations, by Weinan E, Jiequn Han and Arnulf Jentzen,
3. Deep optimal stopping, by S. Becker, P. Cheridito, and A. Jentzen
4. Deep fictitious play for stochastic differential games, by Ruimeng Hu
5. Convergence of deep fictitious play for stochastic differential games, by Jiequn Han, Ruimeng Hu, Jihao Long,
6. Fictitious play for mean-field games: Continuous-time analysis and applications, by Sarah Perrin, Julien Pérolat, Mathieu Laurière, Matthieu Geist, Romuald Elie, Olivier Pietquin
7. Convergence Analysis of Machine Learning Algorithms for the Numerical Solution of Mean Field Control and Games: II - The Finite Horizon Case, by René Carmona, Mathieu Laurière

And three books on deep learning, the theory of stochastic control, and mean-field games:
1. Deep Learning (Adaptive Computation and Machine Learning series) by Ian Goodfellow, Yoshua Bengio, and Aaron Courville
2. Continuous-time Stochastic Control and Optimization with Financial Applications, by Huyen Pham,
3. Probabilistic Theory of Mean Field Games with Applications I & II, by René Carmona and Francois Delarue

MATH 260J (W) Craig, *Optimal Transport in PDE, Geometry, and Applied Mathematics*
Description: Over the past twenty years, optimal transport has emerged as a powerful tool in partial differential equations, geometry, and applied mathematics. While many of these developments are relatively recent, the foundational question of optimal transport is quite old, originally posed by Gaspard Monge in 1781: how can one rearrange a pile of dirt to look like another pile of dirt, using the least amount of effort?

Monge's optimal rearrangements, known as optimal transport plans, naturally induce new ways of interpolating between functions, shedding light on convexity properties of energy functionals arising throughout mathematical physics. In partial differential equations, these convexity properties have led to optimal estimates for stability of solutions and asymptotic behavior. In geometry, these convexity results culminated in a synthetic characterization of Ricci curvature, independent of a manifold's underlying differential structure. In applied mathematics, these results led to a range of new numerical methods for simulating solutions of partial differential equations and computing related optimization problems.

In this topics course, we will introduce the foundations of optimal transport and gradient flows on metric

spaces, and we will discuss applications of these tools in partial differential equations, geometry, and applied mathematics. We will close by considering recent extensions of the optimal transport framework to discrete spaces, which has led to natural formulations of partial differential equations on graphs, discrete notions of Ricci curvature, and new numerical methods.
Prerequisites: Math 201ABC

References:
[1] Gradient Flows in Metric Spaces and the Space of Probability Measures, Ambrosio, Gigli, and Savaré
[2] Entropic Ricci Curvature for Discrete Spaces, Jan Maas
[3] Topics in Optimal Transportation, Cedric Villani
[4] Optimal Transport, Old and New, Cedric Villani

MATH 260L (S), Harutyunyan, *Introduction to the Calculus of Variations*
While this is an introductory course on classical Calculus of Variations, it will go quite deep in the subject. We will start with classical one dimensional Calculus of Variations and proceed to the so-called vector problems. Four basic convexity conditions in Calc. Var, i.e., Convexity, Rank-One Convexity, Quasiconvexity, and Polyconvexity will be presented and studied in quite detail. A special attention will be taken to the "Quasiconvexity" condition and its relation to existence of minimizers in minimization problems. Some applications to Partial Differential Equations and Materials Science problems will be discussed.

MATH 260Q (S), X. Zhao, *Derived Categories of Coherent Sheaves*
This course is an introduction to the study of derived categories of coherent sheaves on algebraic varieties. After introducing the foundation, we will focus on several concrete examples that are most related to current research interest.

A list of potential topics includes:
1. Basics about derived categories and derived functors.
2. Derived categories of coherent sheaves and Fourier-Mukai transforms.
3. Derived categories of projective spaces and exceptional collections.
4. Derived equivalences between abelian varieties and their duals.
5. Stability conditions on K3 surfaces.
6. Derived equivalences between birational Calabi-Yau threefolds.
7. Homological mirror symmetry for elliptic curves.

Prerequisites: Math 237AB.
We will be working with smooth projective varieties and coherent sheaves. People familiar with projective manifolds and holomorphic vector bundles should be able to follow the course.

The background on triangulated categories and derived categories will be briefly recalled at the beginning of the course. Previously taking Math 236AB will be helpful but not required. The purpose of the course is partially to get an understanding of these abstract constructions via the study of concrete examples.

Some references:
"Fourier-Mukai transforms in algebraic geometry" by Daniel Huybrechts.
"Stability conditions on triangulated categories" by Tom Bridgeland, Ann. Math. 166 (2007), 317–345.
"Stability conditions on K3 surfaces" by Tom Bridgeland, Duke Math. J. 141 (2), 241-291.
"Flops and derived categories" by Tom Bridgeland, Invent. Math. 147 (3), 613-632.

"Categorical mirror symmetry: the elliptic curve" by Alexander Polishchuk and Eric Zaslow, Adv. Theor. Math. Phys. 2 (1998) 443-470.

MATH 501 (F), Garfield, *Teaching Assistant Training*
Consideration of ideas about the process of learning mathematics and discussion of approaches to teaching.