2026-27 GRADUATE COURSE DESCRIPTIONS

MATH 201 A-B-C (FWS), Craig/Garcia-Cervera/S. Tang, ***Real Analysis***
Measure theory and integration. Point set topology. Principles of functional analysis. L^p spaces. The Riesz representation theorem. Topics in real and functional analysis.

MATH 202 A-B-C (FWS), Jacobs/Putinar, ***Complex Analysis***
Analytic functions. Complex integration. Cauchy's theorem. Series and product developments. Entire functions. Conformal mappings. Topics in complex analysis.

MATH 206 A (F), Chandrasekaran, ***Matrix Analysis & Computation***
Graduate level-matrix theory with introduction to matrix computations. SVDs, pseudoinverses, variational characterization of eigenvalues, perturbation theory, direct and interative methods for matrix computations.

MATH 206 B (W), Petzold, ***Numerical Simulation***
Linear multistep methods and Runge-Kutta methods for ordinary differential equations: stability, order and convergence. Stiffness. Differential algebraic equations. Numerical solution of boundary value problems.

MATH 206 C (S), Ceniceros, ***Numerical Solution of Partial Differential Equations - Finite Difference Methods***
Finite difference methods for hyperbolic, parabolic and elliptic PDEs, with application to problems in science and engineering. Convergence, consistency, order and stability of finite difference methods. Dissipation and dispersion. Finite volume methods. Software design and adaptivity.

MATH 206 D (F), Atzberger, ***Numerical Solution of Partial Differential Equations - Finite Element Methods***
Weighted residual and finite element methods for the solution of hyperbolic, parabolic and elliptical partial differential equations, with application to problems in science and engineering. Error estimates. Standard and discontinuous Galerkin methods.

MATH 220 A-B-C (FWS), Yao/Castella/Z. Liu, ***Modern Algebra***
Group theory, ring and module theory, field theory, Galois theory, other topics.

MATH 221 A (F), Bigelow, ***Foundations of Topology***
Metric spaces, topological spaces, continuity, Hausdorff condition, compactness, connectedness, product spaces, quotient spaces. Other topics as time allows.

MATH 221 B (W), Bigelow, ***Homotopy Theory***
Homotopy groups, exact sequences, fiber spaces, covering spaces, van Kampen Theorem.

MATH 221 C (S), McCammond, ***Differential Topology***
Topological manifolds, differentiable manifolds, transversality, tangent bundles, Borsuk-Ulam theorem, orientation and intersection number, Lefschetz fixed point theorem, vector fields.

MATH 225 A (F), Krishna, ***Topics in Number Theory – Class Field Theory***
The main objective of this course will be to introduce the students to topics in algebra and algebraic geometry such as Brauer groups and class field theory. This course will broadly begin with discussion of

the topology of discrete valuation fields. We shall then discuss group cohomology and Galois cohomology of profinite groups, Central simple algebras, and Brauer groups of rings and schemes. We shall compute the Brauer group of local fields. We shall then move on to the class field theory of local and global fields. We shall finally do the class field theory of curves over finite fields.

MATH 225 B (W), Z. Liu, *Topics in Number Theory – Arithmetic of CM elliptic curves*
Two modular forms are said to be congruent modulo pn if their Fourier coefficients are congruent modulo pn. In this course, we will discuss arithmetic applications of such congruences—for instance, how the existence of certain congruences can be used to establish the existence of conjectural p-adic L-functions and to prove lower bounds for Selmer groups. As one of the important tools for these arithmetic applications, we will introduce Hida theory of p-adic families of modular forms.
Prerequisites: Math 220ABC or consent of instructor.

MATH 225 C (S), Castella, *Topics in Number Theory – Congruences among modular forms*
Elliptic curves with complex multiplication are of special interest in number theory due their close link with (explicit) class field theory of imaginary quadratic fields. It is for this class of curves that the first cases of the Tate–Shafarevich conjecture for elliptic curves over number fields F was proved by Rubin in the 1980s. The goal of this course is to explain the ingredients that went into Rubin's proof for F = Q, and discuss recent progress for more general number fields.
Prerequisites: A first course in algebraic number theory and basics of elliptic curves.

MATH 227 B (W), Z. Wang, *Advanced Topics in Geometric and Algebraic Topology*
TBA

MATH 232 A-B (FW), McCammond/Bigelow, *Algebraic Topology*
Singular homology and cohomology, exact sequences, Hurewicz theorem, Poincare duality.

MATH 236 A-B (FW), X. Zhao
Algebraic construction of homology and cohomology theories, aimed at applications to topology, geometry, groups and rings. Special emphasis on hom and tensor functors; projective, injective and flat modules; exact sequences; chain complexes; derived functors, in particular, ext and tor.

We will introduce a categorical approach to work with complexes and cohomology. For an abelian category (e.g. category of modules over a ring, category of sheaves of abelian groups over a topological space, etc), we construct its derived category. Roughly speaking, the derived category consists of complexes of objects in the abelian category, and two complexes having the same cohomology are now considered isomorphic to each other. The general framework of triangulated categories and localizations will be introduced to construct such a category and to study its properties. The second part of the course introduces derived functor. We will use sheaves on topological spaces as one major example, the basic definitions will be covered in class.

MATH 240 A-B-C (FWS), Dai/Wei/Ye, *Introduction to Differential Geometry and Riemannian Geometry*
Topics include geometry of surfaces, manifolds, differential forms, Lie groups, Riemannian manifolds, Levi-Civita connection and curvature, curvature and topology, Hodge theory. Additional topics such as bundles and characteristic classes, spin structures Dirac operator, comparison theorems in Riemannian geometry.

MATH 241 A (F), Wei, ***Topics in Differential Geometry - Spaces with Weak Ricci Curvature Lower Bound***
We will focus on geometry, topology and analysis of spaces with lower Ricci curvature bound in weak sense, like integral Ricci, Kato condition, spectral Ricci, RCD spaces.

MATH 241 B (W), Wink, ***Topics in Differential Geometry - Modern topics in the Bochner technique***
Bochner proved that closed manifolds with positive Ricci curvature have vanishing first Betti number. The principle behind the proof is now known as the Bochner technique and provides a deep connection between the curvature and the topology of a Riemannian manifold. This course discusses recent advances in the subject. Along the way we'll discuss topics such as holonomy, symmetric spaces, or Kähler manifolds.

MATH 241 C (S), Dai, ***Topics in Differential Geometry - An Introduction to Atiyah-Singer Index Theorem and its Applications***
Atiyah-Singer index theorem is truly one of the great landmarks of twentieth century mathematics, a grand unification of the classical Gauss-Bonnet formula, the Riemann-Roch formula and Hirzebruch's signature formula, with broad applications in differential geometry, topology, algebraic geometry, representation theory, number theory, and physics. We will present an introduction to the theorem and its various applications.

MATH 246 A-B-C (FWS), Labutin/H. Zhou/Harutyunyan, ***Partial Differential Equations***
Existence and stability of solutions, Floquet theory, Poincare-Bendixson theorem, invariant manifolds, existence and stability of periodic solutions, Bifurcation theory and normal forms, hyperbolic structure and chaos, Feigenbaum period-doubling cascade, Ruelle-Takens cascade.

MATH 260AA (W), Harutyunyan, ***Introduction to the Calculus of Variations***
Convexity conditions, existence and regularity of minimizers of variational problems.

MATH 260CC (F), Z. Yao, ***Introduction to Mathematical Inverse Problems***
In inverse problems one attempts to determine the interior properties of a medium by applying various non-intrusive methods. The mathematical problems under study are often motivated by real world application purposes, including questions arising in medical imaging (e.g. CT, EIT, MRI), geophysics (seismology), mathematical physics, etc. In this topics course, we expect to introduce the mathematical analysis of some basic types of inverse problems, including X-ray and Radon transforms, inverse boundary value problems for PDEs (e.g. the Calderon problem). If time permits, we will also give a brief introduction to related inverse problems, such as travel time tomography, invisibility and cloaking, inverse problems in non-trivial geometry. There are no prerequisites, but some basic knowledge of PDEs and Fourier analysis will be helpful for taking the course.

References: [1] Lecture notes on Calderon problem, Mikko Salo, Spring 2008.
[2] Microlocal Analysis and Integral Geometry, Plamen Stefanov and Gunther Uhlmann (draft)
[3] Inverse problems: visibility and invisibility, Gunther Uhlmann, JEDP 2012.

MATH 260EE (FWS) ???, ***Graduate Student Colloquium***
Topics in algebra, analysis, applied mathematics, combinatorial mathematics, functional analysis, geometry, statistics, topology, by means of lectures and informal conferences with members of faculty.

MATH 260F (W), Atzberger, *Optimization Theory and Applications*
Many problems in decision making and analysis can be formulated as the minimization or maximization of objective functions, along with constraints on permissible solutions. The field of optimization is concerned with developing mathematical approaches to make precise these types of problems, their well-posedness, and how to develop practical numerical methods to approximate their solutions. This course will cover mathematical topics in optimization theory, including in the linear and non-linear settings. The course will also cover motivating applications from the sciences, engineering, and machine learning.

The parts of the course on background will use the books:

- "An Introduction to Optimization with Applications in Machine Learning", (5th edition), E. Chong, W. Lu, and S. Żak
- "Numerical Optimization," by Nocedal and Wright
- "Convex Analysis and Nonlinear Optimization," J. Borwein and A. Lewis.

The special topics part of the course will be based on materials developed by the instructor and recent papers in the literature.

Sample of topics includes: Constrained Optimization, Convex Analysis and Duality, Linear Programming, Interior Point Methods, Non-Linear Optimization, Stochastic Approximation of Objectives, Stochastic Gradient Descent, Gradient Methods with Projections, Proximal Methods, Variational Approximation of PDEs, Machine Learning Approaches, Deep Neural Networks, SGD Training Approaches, and other applications.

MATH 260H (W), S. Tang, *Introduction to High-Dimensional Probability, Approximation and Learning Theory*
The course covers fundamental mathematical ideas for approximation and statistical learning problems in high dimensions. We will start with studying high-dimensional phenomena, through both the probability (concentration inequalities) and geometry (concentration phenomena). We then consider a variety of techniques and problems in high dimensional statistics, machine learning, and signal processing, ranging from dimensional reduction to classification and regression(with connections to approximation theory, Fourier analysis and wavelets, Reproducing kernel Hilbert spaces), to compressed sensing and matrix completion problems. Finally we discuss problems at the intersection of statistical estimation, machine learning and dynamical systems, in particular, the interacting particle system.

Reference list:

1. High dimensional probability: An introduction with applications in data science. R. Vershynin. Main textbook
2. High dimensional statistics. P. Rigollet
3. Ten lectures and Forty two open problems in the mathematics of data science
4. A distribution free theory of nonparametric regression
5. Lectures on spectral graph theory
6. Nonparametric inference of interaction laws in system of agents from trajectory data. by Maggioni et al PNAS.

MATH 260J (F) H. Zhou, *Introduction to Mathematical Inverse Problems*

In inverse problems one attempts to determine the interior properties of a medium by applying various non-intrusive methods. The mathematical problems under study are often motivated by real world application purposes, including questions arising in medical imaging (e.g. CT, EIT, MRI), geophysics (seismology), mathematical physics, etc. In this topics course, we expect to introduce the mathematical analysis of some basic types of inverse problems, including X-ray and Radon transforms, inverse boundary value problems for PDEs (e.g. the Calderon problem). If time permits, we will also give a brief introduction to related inverse problems, such as travel time tomography, invisibility and cloaking, inverse problems in non-trivial geometry. There are no prerequisites, but some basic knowledge of PDEs and Fourier analysis will be helpful for taking the course.

References: [1] Lecture notes on Calderon problem, Mikko Salo, Spring 2008.
[2] Microlocal Analysis and Integral Geometry, Plamen Stefanov and Gunther Uhlmann (draft)
[3] Inverse problems: visibility and invisibility, Gunther Uhlmann, JEDP 2012.

MATH 260L (S), Birnir, *Stochastic ODEs and PDEs*

We develop the theory of stochastic ODEs and PDEs, using the books by Oksendal and Oksendal and Sulem. The applications are to turbulence theory.

MATH 260N (S), R. Hu, *Stochastic Control and Game Theory: Strategic Interference and Adversarial Learning*

This course introduces the mathematical and algorithmic foundations of strategic inference and adversarial decision-making in stochastic control and game-theoretic systems. It explores how autonomous agents learn, infer, and strategically reveal or conceal information in competitive environments.

We develop continuous-time formulations of many-player and mean-field games, emphasizing deception, counter-deception, and intent inference in uncertain, information-asymmetric settings. Topics include stochastic optimal control, partially observed systems, inverse reinforcement learning, and information-design problems. Algorithmic components connect these theories with recent advances in deep learning and reinforcement learning, covering neural approximations of value functions and policies, actor–critic and policy gradient methods, generative modeling for belief and strategy inference, and representation learning techniques such as score-based diffusion models and path signatures for stochastic dynamics.

Reference:
[1] Strategic Inference in Stackelberg Games: Optimal Control for Revealing Adversary Intent, R. Hu, D. Ralston, X. Yang and H. Zhou, 2025
[2] Recent Developments in Machine Learning Methods for Stochastic Control and Games, R. Hu and M. Lauriere, 2024
[3] Deceptive sequential decision-making via regularized policy optimization, M. Hale et al. 2025
[4] Stochastic differential equations, B. Øksendal, 2003
[5] Continuous-time stochastic control and optimization with financial applications, H.Pham, 2009
[6] Lecture notes on "mean field games and interacting diffusion models", D. Lacker, 2018

MATH 260Q (S), Krishna, *Introduction to motivic cohomology*
The aim of this course is to introduce the students to the theory of motivic cohomology, algebraic cycles and their relation to algebraic K-theory. We shall begin with recalling the definition of algebraic cycles from Fulton's book. We shall then study the theory of higher Chow groups by Bloch, followed by the discussion of Suslin–Voevodsky motivic cohomology. We shall show the equivalence of motivic cohomology and higher Chow groups and end with the discussion of the relation between motivic cohomology and algebraic K-theory.

MATH 501 (F), Garfield, *Teaching Assistant Training*
Consideration of ideas about the process of learning mathematics and discussion of approaches to teaching.