Math 7H: Honors Seminar — Professor: Padraic Bartlett

Homework 1: Sizes of Infinity

Due Tuesday, week 2, at the start of class — *UCSB 2014*

Checkdown problem.

1. In class, we defined what it means for a function from $\mathbb{N} \to \mathbb{N}$ to be injective.

   (a) Create two distinct functions $f, g : \mathbb{N} \to \mathbb{N}$, that are both injective. Create a third function $h : \mathbb{N} \to \mathbb{N}$ that is not injective.

   (b) Given two functions f, g, we can form their **composition**, $f \circ g$, as the function formed by first applying g and then f to any input. For example, if $g(x) = x^2$ and $f(x) = x + 1$, the function $f \circ g(x)$ is just $x^2 + 1$.

   Take the three functions f, g, h that you created in part a. Is the composition $f \circ g$ an injective function? How about $f \circ h$?

Extra-credit problems.

2. Can there ever be more words than numbers?

   Specifically: let's suppose that we're limiting ourselves to the 26-character Latin alphabet, and that the only kinds of things that can be **words** are finite strings of characters from the Latin alphabet. So things like

   - rabbit
   - bargleargleSnarg
   - ssss
   - froyo

   are all possibly words. Call the set of all possible words $\mathbb{W}$. Is the set $\mathbb{W}$ the same cardinality as $\mathbb{N}$? Prove your claim.