FORM GR9768

68

THE GRADUATE RECORD EXAMINATIONS®

GRE®

MATHEMATICS TEST (RESCALED)

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MATHEMATICS TEST (RESCALED)

Time—170 minutes

66 Questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. In each case, select the one that is the best of the choices offered and then mark the corresponding space on the answer sheet.

Computation and scratchwork may be done in this examination book.

Note: In this examination:

(1) All logarithms with an unspecified base are natural logarithms (that is, with base e).
(2) The set of all real numbers x such that $a \leq x \leq b$ is denoted by $[a, b]$.
(3) The symbols $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, and $\mathbb{C}$ denote the sets of integers, rational numbers, real numbers, and complex numbers, respectively.

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1. If $F(x) = \int_e^x \log t \, dt$ for all positive x, then $F'(x) =$

(A) x

(B) $\frac{1}{x}$

(C) $\log x$

(D) $x \log x$

(E) $x \log x - 1$

---

2. If $F(1) = 2$ and $F(n) = F(n-1) + \frac{1}{2}$ for all integers $n > 1$, then $F(101) =$

(A) 49
(B) 50
(C) 51
(D) 52
(E) 53

GO ON TO THE NEXT PAGE.

SCRATCHWORK

3. If $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ is invertible under matrix multiplication, then its inverse is

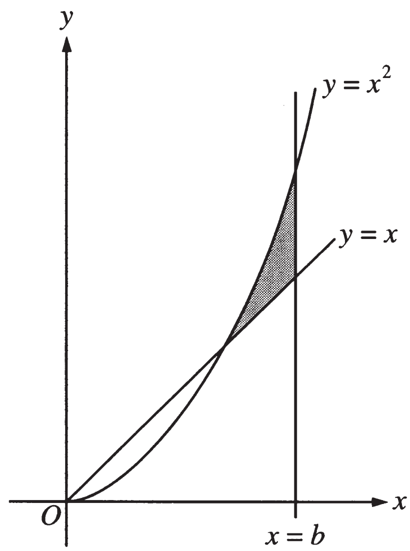
(A) $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

(B) $\dfrac{1}{a^2 + b^2}\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

(C) $\dfrac{1}{a^2 + b^2}\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

(D) $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

(E) $\dfrac{1}{a^2 - b^2}\begin{pmatrix} -b & a \\ a & b \end{pmatrix}$



4. If $b > 0$ and if $\int_0^b x\,dx = \int_0^b x^2\,dx$, then the area of the shaded region in the figure above is

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) $\frac{1}{2}$

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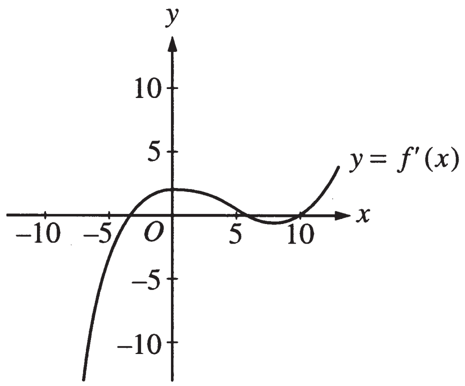
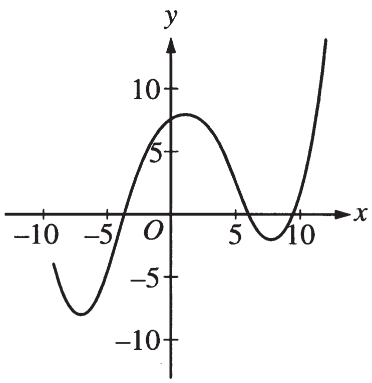
SCRATCHWORK



5. If the figure above is the graph of $y = f'(x)$, which of the following could be the graph of $y = f(x)$?

(A)



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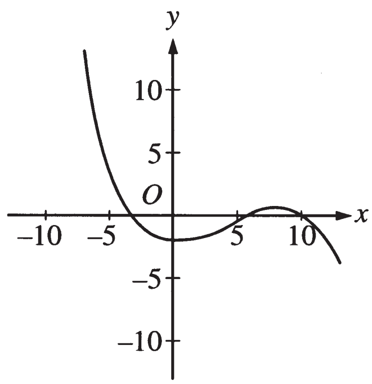

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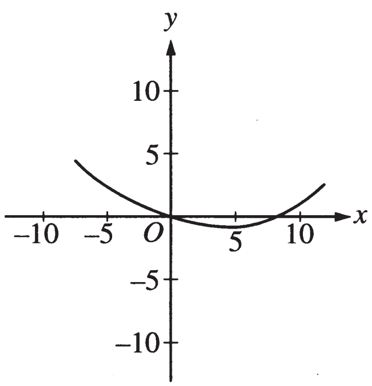

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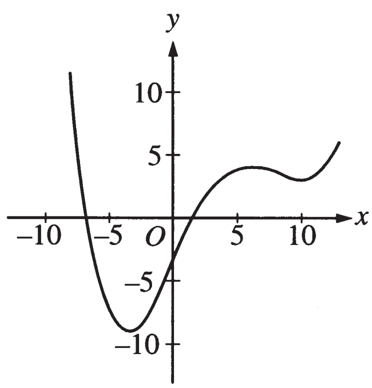
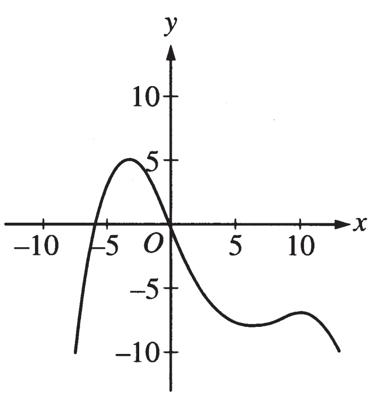


(C)



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SCRATCHWORK

6. Consider the following sequence of instructions.

   1. Set $k = 999$, $i = 1$, and $p = 0$.
   2. If $k > i$, then go to step 3; otherwise go to step 5.
   3. Replace i with $2i$ and replace p with $p + 1$.
   4. Go to step 2.
   5. Print p.

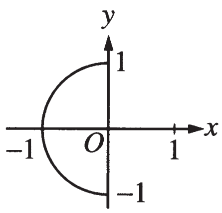
   If these instructions are followed, what number will be printed at step 5 ?

   (A) 1
   (B) 2
   (C) 10
   (D) 512
   (E) 999

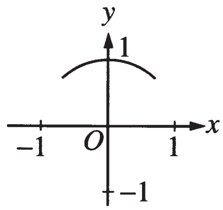
---

7. Which of the following indicates the graph of $\left\{(\sin t, \cos t)\colon -\frac{\pi}{2} \le t \le 0\right\}$ in the xy-plane?

(A)



(D)



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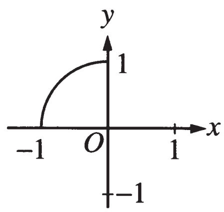

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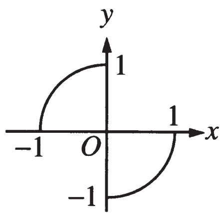
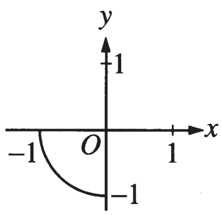


(C)



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SCRATCHWORK

8. $\displaystyle\int_0^1 \frac{x}{1 + x^2}\, dx =$

(A) 1

(B) $\dfrac{\pi}{4}$

(C) $\tan^{-1} \dfrac{\sqrt{2}}{2}$

(D) $\log 2$

(E) $\log \sqrt{2}$

---

9. If S is a nonempty finite set with k elements, then the number of one-to-one functions from S onto S is

(A) $k!$
(B) k^2
(C) k^k
(D) 2^k
(E) 2^{k+1}

---

10. Let g be the function defined on the set of all real numbers by

$$g(x) = \begin{cases} 1 & \text{if } x \text{ is rational,} \\ e^x & \text{if } x \text{ is irrational.} \end{cases}$$

Then the set of numbers at which g is continuous is

(A) the empty set
(B) $\{0\}$
(C) $\{1\}$
(D) the set of rational numbers
(E) the set of irrational numbers

---

11. For all real numbers x and y, the expression $\dfrac{x + y + |x - y|}{2}$ is equal to

(A) the maximum of x and y

(B) the minimum of x and y

(C) $|x + y|$

(D) the average of $|x|$ and $|y|$

(E) the average of $|x + y|$ and $x - y$

GO ON TO THE NEXT PAGE.

SCRATCHWORK

12. Let B be a nonempty bounded set of real numbers and let b be the least upper bound of B. If b is not a member of B, which of the following is necessarily true?

(A) B is closed.
(B) B is not open.
(C) b is a limit point of B.
(D) No sequence in B converges to b.
(E) There is an open interval containing b but containing no point of B.

---

13. A drawer contains 2 blue, 4 red, and 2 yellow socks. If 2 socks are to be randomly selected from the drawer, what is the probability that they will be the same color?

(A) $\frac{2}{7}$

(B) $\frac{2}{5}$

(C) $\frac{3}{7}$

(D) $\frac{1}{2}$

(E) $\frac{3}{5}$

---

14. Let $\mathbb{R}$ be the set of real numbers and let f and g be functions from $\mathbb{R}$ into $\mathbb{R}$. The negation of the statement

"For each s in $\mathbb{R}$, there exists an r in $\mathbb{R}$ such that if $f(r) > 0$, then $g(s) > 0$."

is which of the following?

(A) For each s in $\mathbb{R}$, there does not exist an r in $\mathbb{R}$ such that if $f(r) > 0$, then $g(s) > 0$.
(B) For each s in $\mathbb{R}$, there exists an r in $\mathbb{R}$ such that $f(r) > 0$ and $g(s) \leq 0$.
(C) There exists an s in $\mathbb{R}$ such that for each r in $\mathbb{R}$, $f(r) > 0$ and $g(s) \leq 0$.
(D) There exists an s in $\mathbb{R}$ and there exists an r in $\mathbb{R}$ such that $f(r) \leq 0$ and $g(s) \leq 0$.
(E) For each r in $\mathbb{R}$, there exists an s in $\mathbb{R}$ such that $f(r) \leq 0$ and $g(s) \leq 0$.

---

15. If g is a function defined on the open interval (a, b) such that $a < g(x) < x$ for all $x \in (a, b)$, then g is

(A) an unbounded function
(B) a nonconstant function
(C) a nonnegative function
(D) a strictly increasing function
(E) a polynomial function of degree 1

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SCRATCHWORK

16. For what value (or values) of m is the vector $(1, 2, m, 5)$ a linear combination of the vectors $(0, 1, 1, 1)$, $(0, 0, 0, 1)$, and $(1, 1, 2, 0)$?

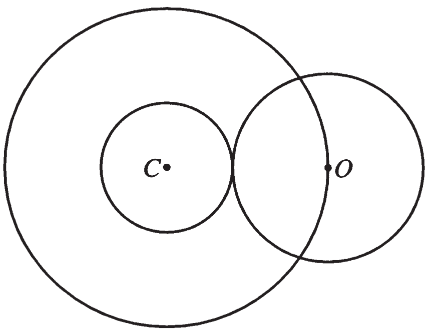
(A) For no value of m
(B) -1 only
(C) 1 only
(D) 3 only
(E) For infinitely many values of m

---

17. For a function f, the finite differences $\Delta f(x)$ and $\Delta^2 f(x)$ are defined by $\Delta f(x) = f(x+1) - f(x)$ and $\Delta^2 f(x) = \Delta f(x+1) - \Delta f(x)$. What is the value of $f(4)$, given the following partially completed finite difference table?

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
1	-1	4	
2		-2	6
3			
4			

(A) -5
(B) -1
(C) 1
(D) 3
(E) 5



18. In the figure above, the annulus with center C has inner radius r and outer radius 1. As r increases, the circle with center O contracts and remains tangent to the inner circle. If $A(r)$ is the area of the annulus and $a(r)$ is the area of the circular region with center O, then $\lim_{r \to 1^-} \frac{A(r)}{a(r)} =$

(A) 0
(B) $\frac{2}{\pi}$
(C) 1
(D) $\frac{\pi}{2}$
(E) ∞

GO ON TO THE NEXT PAGE.

SCRATCHWORK

19. Which of the following are multiplication tables for groups with four elements?

I.

	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

II.

	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	a
d	d	c	a	b

III.

	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	c	d
d	d	c	d	c

(A) None
(B) I only
(C) I and II only
(D) II and III only
(E) I, II, and III

---

20. Which of the following statements are true for every function f, defined on the set of all real numbers, such that $\lim\limits_{x \to 0} \dfrac{f(x)}{x}$ is a real number L and $f(0) = 0$?

I. f is differentiable at 0.

II. $L = 0$

III. $\lim\limits_{x \to 0} f(x) = 0$

(A) None
(B) I only
(C) III only
(D) I and III only
(E) I, II, and III

---

21. What is the area of the region bounded by the coordinate axes and the line tangent to the graph of $y = \frac{1}{8}x^2 + \frac{1}{2}x + 1$ at the point $(0, 1)$?

(A) $\dfrac{1}{16}$

(B) $\dfrac{1}{8}$

(C) $\dfrac{1}{4}$

(D) 1

(E) 2

GO ON TO THE NEXT PAGE.

SCRATCHWORK

22. Let $\mathbb{Z}$ be the group of all integers under the operation of addition. Which of the following subsets of $\mathbb{Z}$ is NOT a subgroup of $\mathbb{Z}$?

(A) $\{0\}$
(B) $\{n \in \mathbb{Z}: n \geq 0\}$
(C) $\{n \in \mathbb{Z}: n \text{ is an even integer}\}$
(D) $\{n \in \mathbb{Z}: n \text{ is divisible by both 6 and 9}\}$
(E) $\mathbb{Z}$

---

23. In the Euclidean plane, point A is on a circle centered at point O, and O is on a circle centered at A. The circles intersect at points B and C. What is the measure of angle BAC ?

(A) 60°
(B) 90°
(C) 120°
(D) 135°
(E) 150°

---

24. Which of the following sets of vectors is a basis for the subspace of Euclidean 4-space consisting of all vectors that are orthogonal to both $(0, 1, 1, 1)$ and $(1, 1, 1, 0)$?

(A) $\{(0, -1, 1, 0)\}$
(B) $\{(1, 0, 0, 0), (0, 0, 0, 1)\}$
(C) $\{(-2, 1, 1, -2), (0, 1, -1, 0)\}$
(D) $\{(1, -1, 0, 1), (-1, 1, 0, -1), (0, 1, -1, 0)\}$
(E) $\{(0, 0, 0, 0), (-1, 1, 0, -1), (0, 1, -1, 0)\}$

---

25. Let f be the function defined by $f(x, y) = 5x - 4y$ on the region in the xy-plane satisfying the inequalities $x \leq 2$, $y \geq 0$, $x + y \geq 1$, and $y - x \leq 0$. The maximum value of f on this region is

(A) 1
(B) 2
(C) 5
(D) 10
(E) 15

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SCRATCHWORK

26. Let f be the function defined by

$$f(x) = \begin{cases} -x^2 + 4x - 2 & \text{if } x < 1, \\ -x^2 + 2 & \text{if } x \geq 1. \end{cases}$$

Which of the following statements about f is true?

(A) f has an absolute maximum at $x = 0$.
(B) f has an absolute maximum at $x = 1$.
(C) f has an absolute maximum at $x = 2$.
(D) f has no absolute maximum.
(E) f has local maxima at both $x = 0$ and $x = 2$.

---

27. Let f be a function such that $f(x) = f(1 - x)$ for all real numbers x. If f is differentiable everywhere, then $f'(0) =$

(A) $f(0)$
(B) $f(1)$
(C) $-f(0)$
(D) $f'(1)$
(E) $-f'(1)$

---

28. If V_1 and V_2 are 6-dimensional subspaces of a 10-dimensional vector space V, what is the smallest possible dimension that $V_1 \cap V_2$ can have?

(A) 0
(B) 1
(C) 2
(D) 4
(E) 6

---

29. Assume that p is a polynomial function on the set of real numbers. If $p(0) = p(2) = 3$ and $p'(0) = p'(2) = -1$, then $\int_0^2 xp''(x)\,dx =$

(A) -3
(B) -2
(C) -1
(D) 1
(E) 2

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SCRATCHWORK

30. Suppose B is a basis for a real vector space V of dimension greater than 1. Which of the following statements could be true?

(A) The zero vector of V is an element of B.
(B) B has a proper subset that spans V.
(C) B is a proper subset of a linearly independent subset of V.
(D) There is a basis for V that is disjoint from B.
(E) One of the vectors in B is a linear combination of the other vectors in B.

---

31. Which of the following CANNOT be a root of a polynomial in x of the form $9x^5 + ax^3 + b$, where a and b are integers?

(A) -9

(B) -5

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) 9

---

32. When 20 children in a classroom line up for lunch, Pat insists on being somewhere ahead of Lynn. If Pat's demand is to be satisfied, in how many ways can the children line up?

(A) 20!

(B) 19!

(C) 18!

(D) $\frac{20!}{2}$

(E) $20 \cdot 19$

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SCRATCHWORK

33. How many integers from 1 to 1,000 are divisible by 30 but not by 16 ?

(A) 29
(B) 31
(C) 32
(D) 33
(E) 38

---

34. Suppose f is a differentiable function for which $\lim_{x\to\infty} f(x)$ and $\lim_{x\to\infty} f'(x)$ both exist and are finite. Which of the following must be true?

(A) $\lim_{x\to\infty} f'(x) = 0$

(B) $\lim_{x\to\infty} f''(x) = 0$

(C) $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} f'(x)$

(D) f is a constant function.

(E) f' is a constant function.

---

35. In xyz-space, an equation of the plane tangent to the surface $z = e^{-x} \sin y$ at the point where $x = 0$ and $y = \frac{\pi}{2}$ is

(A) $x + y = 1$
(B) $x + z = 1$
(C) $x - z = 1$
(D) $y + z = 1$
(E) $y - z = 1$

---

36. For each real number x, let $\mu(x)$ be the mean of the numbers 4, 9, 7, 5, and x; and let $\eta(x)$ be the median of these five numbers. For how many values of x is $\mu(x) = \eta(x)$?

(A) None
(B) One
(C) Two
(D) Three
(E) Infinitely many

GO ON TO THE NEXT PAGE.

SCRATCHWORK

37. $\sum_{k=1}^{\infty} \frac{k^2}{k!} =$

(A) e

(B) $2e$

(C) $(e + 1)(e - 1)$

(D) e^2

(E) ∞

---

38. Which of the following integrals on the interval $\left[0, \frac{\pi}{4}\right]$ has the greatest value?

(A) $\int_0^{\frac{\pi}{4}} \sin t \, dt$

(B) $\int_0^{\frac{\pi}{4}} \cos t \, dt$

(C) $\int_0^{\frac{\pi}{4}} \cos^2 t \, dt$

(D) $\int_0^{\frac{\pi}{4}} \cos 2t \, dt$

(E) $\int_0^{\frac{\pi}{4}} \sin t \cos t \, dt$

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SCRATCHWORK

39. Consider the function f defined by $f(x) = e^{-x}$ on the interval $[0, 10]$. Let $n > 1$ and let $x_0, x_1, \ldots, x_n$ be numbers such that $0 = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = 10$. Which of the following is greatest?

(A) $\sum_{j=1}^{n} f(x_j)(x_j - x_{j-1})$

(B) $\sum_{j=1}^{n} f(x_{j-1})(x_j - x_{j-1})$

(C) $\sum_{j=1}^{n} f\left(\frac{x_j + x_{j-1}}{2}\right)(x_j - x_{j-1})$

(D) $\int_0^{10} f(x)\,dx$

(E) 0

---

40. A fair coin is to be tossed 8 times. What is the probability that more of the tosses will result in heads than will result in tails?

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{87}{256}$

(D) $\frac{23}{64}$

(E) $\frac{93}{256}$

---

41. The function $f(x, y) = xy - x^3 - y^3$ has a relative maximum at the point

(A) $(0, 0)$

(B) $(1, 1)$

(C) $(-1, -1)$

(D) $(1, 3)$

(E) $\left(\frac{1}{3}, \frac{1}{3}\right)$

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SCRATCHWORK

42. Consider the points $A = (-1, 2)$, $B = (6, 4)$, and $C = (1, -20)$ in the plane. For how many different points D in the plane are A, B, C, and D the vertices of a parallelogram?

(A) None
(B) One
(C) Two
(D) Three
(E) Four

---

43. If A is a 3×3 matrix such that $A\begin{pmatrix}0\\1\\2\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$ and $A\begin{pmatrix}3\\4\\5\end{pmatrix} = \begin{pmatrix}0\\1\\0\end{pmatrix}$, then the product $A\begin{pmatrix}6\\7\\8\end{pmatrix}$ is

(A) $\begin{pmatrix}0\\0\\1\end{pmatrix}$

(B) $\begin{pmatrix}-1\\2\\0\end{pmatrix}$

(C) $\begin{pmatrix}1\\-1\\0\end{pmatrix}$

(D) $\begin{pmatrix}9\\10\\11\end{pmatrix}$

(E) not uniquely determined by the information given

---

44. Let f denote the function defined for all $x > 0$ by $f(x) = (\sqrt{x})^x$. Which of the following statements is FALSE?

(A) $\lim\limits_{x \to 0^+} f(x) = 1$

(B) $\lim\limits_{x \to \infty} f(x) = \infty$

(C) $f(x) = x^{x/2}$ for all $x > 0$.

(D) The derivative $f'(x)$ is positive for all $x > 0$.

(E) The derivative $f'(x)$ is increasing for all $x > 0$.

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SCRATCHWORK

45. An experimental car is found to have a fuel efficiency $E(v)$, in miles per gallon of fuel, where v is the speed of the car, in miles per hour. For a certain 4-hour trip, if $v = v(t)$ is the speed of the car t hours after the trip started, which of the following integrals represents the number of gallons of fuel that the car used on the trip?

(A) $\displaystyle\int_0^4 \frac{v(t)}{E(v(t))}\,dt$

(B) $\displaystyle\int_0^4 \frac{E(v(t))}{v(t)}\,dt$

(C) $\displaystyle\int_0^4 \frac{tv(t)}{E(v(t))}\,dt$

(D) $\displaystyle\int_0^4 \frac{tE(v(t))}{v(t)}\,dt$

(E) $\displaystyle\int_0^4 v(t)E(v(t))\,dt$

---

46. For $0 < t < \pi$, the matrix $\begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix}$ has distinct complex eigenvalues λ_1 and λ_2. For what value of t, $0 < t < \pi$, is $\lambda_1 + \lambda_2 = 1$?

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

(E) $\frac{2\pi}{3}$

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SCRATCHWORK

47. Let x and y be uniformly distributed, independent random variables on $[0, 1]$. The probability that the distance between x and y is less than $\frac{1}{2}$ is

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

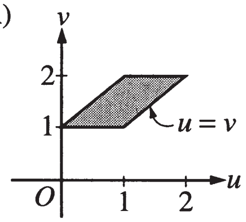
(E) $\frac{3}{4}$

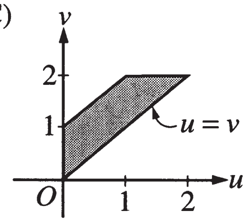
---

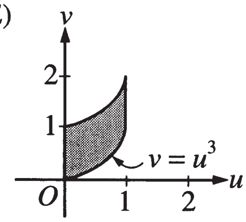
48. Consider the change of variables from the xy-plane to the uv-plane given by the equations

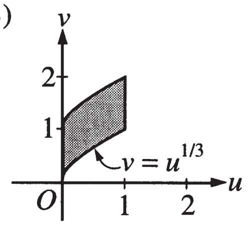
$$u = x^{1/3} + y$$
$$v = 1 + y.$$

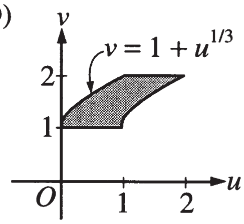
Under this transformation, the image of the region $\{(x, y): 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1\}$ is which of the following shaded regions?

(A)


(C)


(E)


(B)


(D)


GO ON TO THE NEXT PAGE.

SCRATCHWORK

49. If f is a continuous function on the set of real numbers and if a and b are real numbers, which of the following must be true?

I. $\displaystyle\int_a^b f(x)\,dx = \int_{a+3}^{b+3} f(x-3)\,dx$

II. $\displaystyle\int_a^b f(x)\,dx = \int_a^3 f(x)\,dx - \int_b^3 f(x)\,dx$

III. $\displaystyle\int_{3a}^{3b} f(x)\,dx = 3\int_a^b f(3x)\,dx$

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, and III

---

50. How many continuous real-valued functions f are there with domain $[-1, 1]$ such that $(f(x))^2 = x^2$ for each x in $[-1, 1]$?

(A) One
(B) Two
(C) Three
(D) Four
(E) Infinitely many

---

51. Let D be the region in the xy-plane in which the series $\displaystyle\sum_{k=1}^{\infty} \frac{(x+2y)^k}{k}$ converges. Then the interior of D is

(A) an open disk
(B) the open region bounded by an ellipse
(C) the open region bounded by a quadrilateral
(D) the open region between two parallel lines
(E) an open half plane

GO ON TO THE NEXT PAGE.

SCRATCHWORK

52. Consider the following system of linear equations over the real numbers, where x, y, and z are variables and b is a real constant.

$$\begin{aligned} x + y + z &= 0 \\ x + 2y + 3z &= 0 \\ x + 3y + bz &= 0 \end{aligned}$$

Which of the following statements are true?

I. There exists a value of b for which the system has no solution.
II. There exists a value of b for which the system has exactly one solution.
III. There exists a value of b for which the system has more than one solution.

(A) II only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

---

53. In the complex plane, let C be the circle $|z| = 2$ with positive (counterclockwise) orientation. Then $\int_C \frac{dz}{(z-1)(z+3)^2} =$

(A) 0

(B) $2\pi i$

(C) $\frac{\pi i}{2}$

(D) $\frac{\pi i}{8}$

(E) $\frac{\pi i}{16}$

---

54. The inside of a certain water tank is a cube measuring 10 feet on each edge and having vertical sides and no top. Let $h(t)$ denote the water level, in feet, above the floor of the tank at time t seconds. Starting at time $t = 0$, water pours into the tank at a constant rate of 1 cubic foot per second, and simultaneously, water is removed from the tank at a rate of $0.25h(t)$ cubic feet per second. As $t \to \infty$, what is the limit of the volume of the water in the tank?

(A) 400 cubic feet
(B) 600 cubic feet
(C) 1,000 cubic feet
(D) The limit does not exist.
(E) The limit exists, but it cannot be determined without knowing $h(0)$.

GO ON TO THE NEXT PAGE.

SCRATCHWORK

55. Suppose that f is a twice-differentiable function on the set of real numbers and that $f(0)$, $f'(0)$, and $f''(0)$ are all negative. Suppose f'' has all three of the following properties.

I. It is increasing on the interval $[0, \infty)$.
II. It has a unique zero in the interval $[0, \infty)$.
III. It is unbounded on the interval $[0, \infty)$.

Which of the same three properties does f necessarily have?

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

---

56. For every set S and every metric d on S, which of the following is a metric on S ?

(A) $4 + d$
(B) $e^d - 1$
(C) $d - |d|$
(D) d^2
(E) $\sqrt{d}$

---

57. Let $\mathbb{R}$ be the field of real numbers and $\mathbb{R}[x]$ the ring of polynomials in x with coefficients in $\mathbb{R}$. Which of the following subsets of $\mathbb{R}[x]$ is a subring of $\mathbb{R}[x]$?

I. All polynomials whose coefficient of x is zero
II. All polynomials whose degree is an even integer, together with the zero polynomial
III. All polynomials whose coefficients are rational numbers

(A) I only
(B) II only
(C) I and III only
(D) II and III only
(E) I, II, and III

---

58. Let f be a real-valued function defined and continuous on the set of real numbers $\mathbb{R}$. Which of the following must be true of the set $S = \{f(c)\colon\ 0 < c < 1\}$?

I. S is a connected subset of $\mathbb{R}$.
II. S is an open subset of $\mathbb{R}$.
III. S is a bounded subset of $\mathbb{R}$.

(A) I only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

GO ON TO THE NEXT PAGE.

SCRATCHWORK

59. A cyclic group of order 15 has an element x such that the set $\{x^3, x^5, x^9\}$ has exactly two elements. The number of elements in the set $\{x^{13n}: n \text{ is a positive integer}\}$ is

(A) 3
(B) 5
(C) 8
(D) 15
(E) infinite

---

60. If S is a ring with the property that $s = s^2$ for each $s \in S$, which of the following must be true?

I. $s + s = 0$ for each $s \in S$.
II. $(s + t)^2 = s^2 + t^2$ for each $s, t \in S$.
III. S is commutative.

(A) III only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

---

61. What is the greatest integer that divides $p^4 - 1$ for every prime number p greater than 5 ?

(A) 12
(B) 30
(C) 48
(D) 120
(E) 240

---

62. The coefficient of x^3 in the expansion of $(1 + x)^3(2 + x^2)^{10}$ is

(A) 2^{14}

(B) 31

(C) $\binom{3}{3} + \binom{10}{1}$

(D) $\binom{3}{3} + 2\binom{10}{1}$

(E) $\binom{3}{3}\binom{10}{1}2^9$

GO ON TO THE NEXT PAGE.

SCRATCHWORK

63. At how many points in the xy-plane do the graphs of $y = x^{12}$ and $y = 2^x$ intersect?

(A) None
(B) One
(C) Two
(D) Three
(E) Four

---

64. Suppose that f is a continuous real-valued function defined on the closed interval $[0, 1]$. Which of the following must be true?

I. There is a constant $C > 0$ such that $|f(x) - f(y)| \le C$ for all x and y in $[0, 1]$.

II. There is a constant $D > 0$ such that $|f(x) - f(y)| \le 1$ for all x and y in $[0, 1]$ that satisfy $|x - y| \le D$.

III. There is a constant $E > 0$ such that $|f(x) - f(y)| \le E|x - y|$ for all x and y in $[0, 1]$.

(A) I only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

---

65. Let $p(x)$ be the polynomial $x^3 + ax^2 + bx + c$, where a, b, and c are real constants. If $p(-3) = p(2) = 0$ and $p'(-3) < 0$, which of the following is a possible value of c ?

(A) -27

(B) -18

(C) -6

(D) -3

(E) $-\frac{1}{2}$

---

66. In the xy-plane, if C is the circle $x^2 + y^2 = 9$, oriented counterclockwise, then $\oint_C -2y\,dx + x^2\,dy =$

(A) 0
(B) 6π
(C) 9π
(D) 12π
(E) 18π

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS TEST.

SCRATCHWORK

DO NOT USE INK

Use only a pencil with soft, black lead (No. 2 or HB) to complete this answer sheet. Be sure to fill in completely the space that corresponds to your answer choice. Completely erase any errors or stray marks.

GRADUATE RECORD EXAMINATIONS® - GRE® - SUBJECT TEST

SIDE 1

1. NAME Enter your last name, first name initial (given name), and middle initial if you have one. Omit spaces, apostrophes, Jr., II., etc.

Last Name only (Family or Surname) - first 15 letters | First Name Initial | Middle Initial

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

2.

YOUR NAME: ____________________
(Print) Last Name (Family or Surname) First Name (Given) M.I.

MAILING ADDRESS: ____________________
(Print) P.O. Box or Street Address

City State or Province

Country Zip or Postal Code

CENTER: ____________________
City State or Province

Country Center Number Room Number

SIGNATURE: ____________________

BE SURE EACH MARK IS DARK AND COMPLETELY FILLS THE INTENDED SPACE AS ILLUSTRATED HERE: ●

YOU MAY FIND MORE RESPONSE SPACES THAN YOU NEED. IF SO, PLEASE LEAVE THEM BLANK.

1	A B C D E	39	A B C D E	77	A B C D E
2	A B C D E	40	A B C D E	78	A B C D E
3	A B C D E	41	A B C D E	79	A B C D E
4	A B C D E	42	A B C D E	80	A B C D E
5	A B C D E	43	A B C D E	81	A B C D E
6	A B C D E	44	A B C D E	82	A B C D E
7	A B C D E	45	A B C D E	83	A B C D E
8	A B C D E	46	A B C D E	84	A B C D E
9	A B C D E	47	A B C D E	85	A B C D E
10	A B C D E	48	A B C D E	86	A B C D E
11	A B C D E	49	A B C D E	87	A B C D E
12	A B C D E	50	A B C D E	88	A B C D E
13	A B C D E	51	A B C D E	89	A B C D E
14	A B C D E	52	A B C D E	90	A B C D E
15	A B C D E	53	A B C D E	91	A B C D E
16	A B C D E	54	A B C D E	92	A B C D E
17	A B C D E	55	A B C D E	93	A B C D E
18	A B C D E	56	A B C D E	94	A B C D E
19	A B C D E	57	A B C D E	95	A B C D E
20	A B C D E	58	A B C D E	96	A B C D E
21	A B C D E	59	A B C D E	97	A B C D E
22	A B C D E	60	A B C D E	98	A B C D E
23	A B C D E	61	A B C D E	99	A B C D E
24	A B C D E	62	A B C D E	100	A B C D E
25	A B C D E	63	A B C D E	101	A B C D E
26	A B C D E	64	A B C D E	102	A B C D E
27	A B C D E	65	A B C D E	103	A B C D E
28	A B C D E	66	A B C D E	104	A B C D E
29	A B C D E	67	A B C D E	105	A B C D E
30	A B C D E	68	A B C D E	106	A B C D E
31	A B C D E	69	A B C D E	107	A B C D E
32	A B C D E	70	A B C D E	108	A B C D E
33	A B C D E	71	A B C D E	109	A B C D E
34	A B C D E	72	A B C D E	110	A B C D E
35	A B C D E	73	A B C D E	111	A B C D E
36	A B C D E	74	A B C D E	112	A B C D E
37	A B C D E	75	A B C D E	113	A B C D E
38	A B C D E	76	A B C D E	114	A B C D E

Item responses continued on reverse side.

3. DATE OF BIRTH

Month: Jan. Feb. Mar. April May June July Aug. Sept. Oct. Nov. Dec.
Day: 0 1 2 3 | 0 1 2 3 4 5 6 7 8 9
Year: 0 1 2 3 4 5 6 7 8 | 0 1 2 3 4 5 6 7 8 9

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SIDE 2

SUBJECT TEST

COMPLETE THE CERTIFICATION STATEMENT, THEN TURN ANSWER SHEET OVER TO SIDE 1.

CERTIFICATION STATEMENT

Please write the following statement below, DO NOT PRINT.
"I certify that I am the person whose name appears on this answer sheet. I also agree not to disclose the contents of the test I am taking today to anyone."
Sign and date where indicated.

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SIGNATURE: ____________________________ **DATE:** ___/___/___
Month Day Year

BE SURE EACH MARK IS DARK AND COMPLETELY FILLS THE INTENDED SPACE AS ILLUSTRATED HERE: ⬬.
YOU MAY FIND MORE RESPONSE SPACES THAN YOU NEED. IF SO, PLEASE LEAVE THEM BLANK.

115	A	B	C	D	E	147	A	B	C	D	E	179	A	B	C	D	E	211	A	B	C	D	E
116	A	B	C	D	E	148	A	B	C	D	E	180	A	B	C	D	E	212	A	B	C	D	E
117	A	B	C	D	E	149	A	B	C	D	E	181	A	B	C	D	E	213	A	B	C	D	E
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120	A	B	C	D	E	152	A	B	C	D	E	184	A	B	C	D	E	216	A	B	C	D	E
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143	A	B	C	D	E	175	A	B	C	D	E	207	A	B	C	D	E	239	A	B	C	D	E
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145	A	B	C	D	E	177	A	B	C	D	E	209	A	B	C	D	E	241	A	B	C	D	E
146	A	B	C	D	E	178	A	B	C	D	E	210	A	B	C	D	E	242	A	B	C	D	E

TR	TW	TFS	TCS	1R	1W	1FS	1CS	2R	2W	2FS	2CS
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				5R	5W	5FS	5CS	6R	6W	6FS	6CS

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