Math/CS 120: Intro. to Math | Professor: Padraic Bartlett

Syllabus: Introduction to Higher Mathematics

Weeks 1-10 | *UCSB 2014*

Basic Course Information

- Professor: Padraic Bartlett.
- Email: padraic@math.ucsb.edu.
- Class time/location: MWF 2-3:20, CCS Building 494, Room 164B.
- Office hours/location: 12-1pm Tuesday, SH 6516 / 5-6:30pm Thursday, SH 6516 / 10:30-12pm Saturday, CCS Building 494, Room 164B. These office hours will be shared with other classes. Also, the Saturday hours might not run every weekend, due to travel commitments; I'll email changes when relevant. Also-also, the Saturday hours will have pancakes! Yay, pancakes.
- Course webpage: http://math.ucsb.edu/~padraic/.
- Homework due date: Fridays, at the start of class.

Course Description

Every major field of study in academia, roughly speaking has a way of "showing" that something is true. In English/critical literature studies, if you wanted to argue that the color white in Melville's Moby Dick was intrinsically tied up with mortality, you would write an essay that quoted Melville's epic story alongside some of of his other writings and perhaps some contemporary literature, and logically argue (using these quotations as "evidence") that your claim holds. Similarly, if you were a physicist and you wanted to demonstrate Bell's inequality, which roughly states that local realism and quantum mechanics are incompatible theories, you'd create an experiment under which these two theories necessarily predict different outcomes.

In mathematics, a **proof** is an argument we use as mathematicians to show that something is true. In this class, we're going to study what proofs are.

To do this, our course is going to have a slightly different feel than most other classes you've had. Specifically: this course is going to be focused as much on the way arguments are formed as on the mathematics that we're studying. Unlike other math classes, where we are typically aiming towards a major theorem or idea throughout the class, this course is focused on mathematics as a language; it's designed to be a place where we practice writing proofs and thinking like mathematicians. As in my other CCS classes, homework problems will be graded according to two rubrics:

1. Did you arrive at the correct answer, using a sound chain of logical statements?

2. Does your work cleanly and carefully lay out a full and complete solution to the problem at hand? I.e. could your solution be used as an example in a textbook?

Each homework problem will receive either a point (if the work satisfies both of the above criteria,) half a point (if it makes substantial progress towards these criteria, but is flawed) or no credit (if it is fundamentally flawed on at least one of the two criteria above.) The TA and I will be pretty strict when grading these problems.

In particular, because this is a proofs course, solutions that do not contain proofs / solutions that contain poor proofs are likely to receive no credit, as the point of many exercises is not just to get the right answer, but rather to be able to prove that your answer is right.

Course Evaluation

There are three components of your grade in this course:

1. **Homework** (75%.) As noted before, there will be daily problem sets, collected and turned in **at the start of class on Friday** each week. Problem sets need to be written in LaTeX to be graded.

   Over the course, your lowest three daily HW scores will be dropped! Correspondingly, to make life easier for the grader and myself, **late homework will not be accepted.** Exceptions to this policy can be made with at least 24 hours notice for students with legitimate reasons (sickness, travel, other reasonable difficulties.) If it is within 24 hours of the deadline, then exceptions will only be granted with a corresponding doctor's/counselor's note. Talk to me if you are confused by this policy, or have any questions.

2. **Quizzes** (25%.) There will be weekly brief quizzes on Fridays at the start of class. Your lowest quiz score will be dropped; accordingly, quizzes cannot be made up (with a similar 24-hour-in-advance buffer required to seek any exceptions.)

3. **Extra Credit** (?%) Before any class starts, you may write on the board any of the problems that you had difficulty with. If this problem is on a homework set that is still live, we'll talk for a bit about how to approach it; students who know how to solve it are invited to offer hints here! If the problem is on a homework set that was handed in (either at the start of this class or a previous class,) any student who feels like they have a solution to any such problem is welcome to present their solution to the class. This is done with the caveat that we may not always be able to get to all of the problems listed in any class, due to time constraints.

   If a student successfully presents a correct solution to a problem in this fashion in a class, they get a flat +.5% to their final grade! In the event that there are multiple students with solutions to a problem, the student with the smallest number of presented problems will get priority. Incorrect or flawed presentations may or may not receive partial credit, depending on the quality of the attempted solution.

This course is pass-fail and for five units. As defined by the CCS Mathematics department, your percentage score in this class is transformed into units via the following policy:

- Students receive the full five units for work at or above the A- level.
- Students receive four units for work at or above the B level.
- Students receive three units for work at or above the C level.
- Students receive no units for work below the C- level.

The correspondence between percentage marks and letter grades depends heavily on class performance; there are years where a 75% can correspond to an A, and others where it corresponds to a C, depending on the pace/difficulty of the course. Throughout the course, I will report HW averages along with what letter grade those performances correspond to, so that you can keep track of your position throughout the class.

However, if you want an absolute metric to keep in mind: a 90% will always correspond to some flavor of A, an 80% will always be at least a B, and so on/so forth. It is likely that our grading scale will be kinder than this.

Collaboration/resources policy

Collaboration is allowed (and indeed encouraged) on the homework sets; mathematics at the research level is a collaborative activity, and there is no reason that it should not also be this way in a classroom. Work with your classmates!

Resources are a little trickier. On one hand, you are now researchers; limiting your resources would seem to be contradictory to the spirit of emulating what research mathematics is like. On the other hand, answers to almost everything in elementary mathematics can be found via Google and some patience; if you had unfettered access to every resources in existence, you would likely inadvertently rob yourself of some of the best problems in your education. So we need to strike a balance.

For this class: Wikipedia is a legitimate resource, as are any physical books you get from the library or have yourself. Mathematica/Wolfram Alpha/etc. are also valid tools, though you need to justify any calculations you perform using any computational systems. Upper-classmen are also valid resources to talk to about problems, provided you follow the citation system described below; however, I would ask that you restrict them to hints instead of answers if they know the problem! (This should rarely happen; these are not problems they would have seen in their own run of this class.) Other resources are off-limits. If this policy seems restrictive, talk to me; I am more than glad to make common-sense exceptions where appropriate.

The only things that we ask of you are the following:

1. Write up your work separately, and only write up solutions you understand fully.
2. When writing up your own work, you can directly cite and use without proof anything proven in class or in the class notes posted online. Anything else — i.e. results from

textbooks, Wikipedia, etc. — you need to both (1) cite in your writeup, and (2) reprove the results you're using from those sources carefully in your own words. Simply copying solutions over directly is plagiarism / cheating / otherwise poor academic form; it is passing of as your own work the ideas of others. You are certainly welcome to read and learn what other people have attempted! All I am asking you to do here is to (1) not pass it off as your own work, and (2) rephrase and present it in a new way so that it is clear that you have actually learned something.

3. As an important corollary to the above: if the TA or I find that you have copied sentences/work/etc. directly from outside sources,

   (a) On your first violation, the offending set's score will be set to -100%.

   (b) If there is any second violation, we will get the university involved. Consequences include failing the course and a likely dismissal from CCS.

   Please, please don't make me have to ever go through (b) above.

4. If you work with other students on a problem, it is considered good form to refer to them (i.e. "I worked with Andrew Wiles on this proof of Fermat's Last Theorem") when writing up your solutions. I mostly ask this because crediting collaborators is something you're going to do as mathematicians, and should get in the habit of.

5. Don't post questions to online messageboard-style services.

If you have any questions on the collaboration policy, please email me and I'll be glad to clarify matters.

Course Textbook

There is no primary textbook for this course. I will post notes on the course website.

Tentative Course Outline

This depends heavily on what pace I can take this course at, and what your respective interests and desires are; it is likely that we will take at least a few detours from this path as the quarter progresses! That said, here is a tentative set of topics for the course:

1. What is a proof?
2. Set theory; injection, surjection, bijection, and Schroeder-Bernstein.
3. Formal logic.
4. Proof methods.
5. Number theory.
6. Set theory: the ordinal hierarchy.
7. Rational and real number systems.
8. Limits.
9. Asymptotics.
10. Probability and proofs.