Math/CS 120: Intro. to Math	Professor: Padraic Bartlett

Homework 8: More Set Theory

Due Friday, Week 4 *UCSB 2014*

Do **one** of the **three** problems below!

1. In class on Monday, for any two sets x, y we defined the **ordered pair** (x, y) to be the two-element set $\{\{x\}, \{x, y\}\}$. We picked this definition because we could uniquely translate any ordered pair (x, y) into such a two-element set, and always decode any such two-element set uniquely into an ordered pair.

   You might hope that a similar construction would work for **ordered triples**: i.e. one might hope that for any sets x, y, z, the definition

   $$(x, y, z) := \big\{\{x\}, \{x, y\}, \{x, y, z\}\big\}$$

   would give us a way to describe ordered triples. Show that this definition is "bad," in the following sense: find objects a, b, c, d, e, f such that $(a, b, c) \neq (d, e, f)$ but $\big\{\{a\}, \{a, b\}, \{a, b, c\}\big\} = \big\{\{d\}, \{d, e\}, \{d, e, f\}\big\}$. (In other words, this definition classifies different ordered triples as the same thing, which is not what we'd want!)

2. Show that there is no set A that contains every ordered pair.

3. Suppose that A, B, C are all sets. Prove or disprove the following statements:

   (a) $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

   (b) If $A \neq \emptyset$ and $A \times B = A \times C$, then $B = C$.

   (c) If D is the set $\{A \times X \mid X \in B\}$, then[1] $A \times (\bigcup B) = \bigcup D$.

[1] On our last problem set, we defined $\bigcup A$ as the union $\bigcup_{A' \in A} A'$ of all of the elements of A.