Math/CS 120: Intro. to Math — Professor: Padraic Bartlett

Homework 2: More Proofs

Due Friday, week 1 — *UCSB 2014*

Do **two** of the **four** problems listed here! Prove all of your claims.

1. Prove or disprove the following statements. If you disprove any statement, include an example that disproves the statement: if you prove a statement, include an example that proves the claim made.

   (a) If x and y are irrational, then $x + y$ is irrational.

   (b) If x is irrational and y is rational, then $x + y$ is irrational.

   (c) If x and y are rational, then $x + y$ is rational.

   (d) If x and y are irrational, then $x \cdot y$ is irrational.

   (e) If x is irrational and y is rational, then $x \cdot y$ is irrational.

   (f) If x and y are rational, then $x \cdot y$ is rational.

   (g) If x and y are rational, then x^y is rational.

2. Suppose that you have a 10×10 chessboard, and that you want to cover it with (1×4)-sized dominoes, so that no dominoes overlap or stick off the board. Can you do this? Or is it impossible? (Prove either claim.)

3. The game of **generalized n-tic-tac-toe** is played as follows: on a $n \times n$ grid, two players X and O take turns placing their respective symbols x, o into cells of the grid. No cell can be repeated. The game ends whenever any player gets n consecutive copies of their symbol on the same row /column / diagonal, or when the grid is completely filled in without any player having any such n consecutive symbols. (Normal tic-tac-toe is where $n = 3$.)

   Prove that there is no strategy in generalized tic-tac-toe where the **second player** to move is guaranteed to win.

4. Prove that there are infinitely many prime numbers.