Math/CS 120: Intro. to Math — Professor: Padraic Bartlett

Homework 18: More Complex Numbers

Due Friday at 11:30am, Finals Week — *UCSB 2014*

Solve **three** of the following **six** problems. Also, this set is **extra-credit!** This set can be submitted by email if you can't turn it in to my office. Have fun!

1. Factor 1500 into primes over the Gaussian integers $\mathbb{Z}[i]$.

2. Prove that there are infinitely many Gaussian integer primes.

3. Let ω denote the complex number $e^{2\pi i/3}$, which on the past HW you proved was a third root of unity (that is, $\omega^3 = 1$). Consider the set $\mathbb{Z}[\omega]$ of the **Eisenstein integers**, defined as follows:

$$\mathbb{Z}[\omega] = \{a + b\omega \mid a, b \in \mathbb{R}\}.$$

   (a) Prove that the Eisenstein integers are closed under multiplication and addition.

   (b) Let $N(a + b\omega)$ denote the square of the distance of $a + b\omega$ from the origin in $\mathbb{C}$. Prove that $N(a + b\omega) = a^2 + b^2 - ab$.

   (c) Show that $\pm 1, \pm\omega, \pm(1 + \omega)$ are the only three Eisenstein integers z for which $N(z) = 1$.

4. Create a version of the Euclidean algorithm that works for the Eisenstein integers. Use it to prove that in the Eisenstein integers, all elements can be factored uniquely into Eisenstein-integer primes.

5. Use the Eisenstein integers to find all integer solutions to the equation $y^2 = x^3 + 1$.

6. Prove that unique factorization does not hold for the Hurwitz integers: that is, find some $\tau = a + bi + cj + dk \in \mathcal{H}$ with two distinct factorizations into different sets of irreducible elements.