

## Homework 16: Linear Algebra and Field Extensions

*Due Friday, Week 9**UCSB 2014*

Solve **one** of the following **three** problems. As always, prove your claims/have fun!

1. In class, we claimed that if  $V$  is a vector space and  $A$  is a subset of  $V$ , then the **span** of  $A$ ,  $\text{span}(A)$ , is a vector space. Prove this claim!
2. In class, we claimed that the collection of all polynomials with real-valued coefficients  $\mathbb{R}[x]$  was a vector space. We verbally talked about why this was true, but didn't do a formal proof.

Write down a formal proof of this claim (i.e. check that this object forms a vector space over  $\mathbb{R}$ !)

3. In class, we made the following claim:

**Theorem.** Suppose that  $V$  is a vector space with two bases  $B_1 = \{\vec{v}_1, \dots, \vec{v}_n\}$ ,  $B_2 = \{\vec{w}_1, \dots, \vec{w}_m\}$  both containing finitely many elements.. Then these sets have the same size: i.e.  $|B_1| = |B_2|$ .

Prove this claim!