

Homework 15: Transcendental Numbers

*Due Monday, Week 9**UCSB 2014*

Solve **one** of the following **four** problems. As always, prove your claims/have fun! Also, this set is **extra-credit**, because of the holiday! Take care, and I'll see you all in a week!
–Paddy

1. Prove that the constant

$$L = \sum_{n=1}^{\infty} 10^{-(n!)}$$

is transcendental.

2. Let A be the collection of all algebraic real numbers. What is the cardinality of A ? What is the measure of A ? (Prove both claims.)
3. A **Liouville number** x is an irrational number with the following property: for any positive $n \in \mathbb{N}$, there is some $p, q \in \mathbb{Z}$, $q > 1$, such that

$$0 < \left| x - \frac{p}{q} \right| < \frac{1}{q^n}.$$

In a sense, Liouville numbers are irrational numbers that can be very closely approximated by rational numbers!

Show that the constant in problem 1 is a Liouville number.

4. Show that any Liouville number is transcendental.