

Homework 10: Revision

*Due Friday, Week 5**UCSB 2014*

This week's homework is a little different than normal! What I want you to do is the following:

1. Go through your old HW and quizzes!
2. Find **two** problems that you attempted, but did not get full credit on. (I.e. 0 or .5 scores on attempted work.)
3. Attach your old work (you should have it still in LaTeX!)
4. Attach a paragraph explaining why your old attempt didn't work / your proof was flawed / etc.
5. Then, **fix** your problem by attaching a new and correct solution!

To illustrate how I'd like you to format your work, I've offered an example here. Download this problem set's .tex (as always, you can do this by replacing the .pdf in any url on my website with .tex) to get the specific LaTeX commands I used here, if you want!

Claim. There are infinitely many prime numbers.

Old proof that does not work: 3's a prime, 5's a prime, 7's a prime... looks true.

Reasons that the old proof does not work: I only gave three examples of prime numbers; this doesn't prove that there are infinitely many primes!

New proof: We proceed by contradiction. Suppose that there were only finitely many prime numbers: take all of them, and label them p_1, \dots, p_n . Consider their product $\prod_{i=1}^n p_i$. This number is 0 mod p_i for every i , as it is a multiple of p_i for every i . This, however, means that $1 + \prod_{i=1}^n p_i$ is 1 mod p_i for any i , and thus in particular not a multiple of any of our earlier primes! However, because $1 + \prod_{i=1}^n p_i$ is a number, it must have a factorization into primes — therefore, our finite list of primes was incomplete, as it is missing $1 + \prod_{i=1}^n p_i$'s factors! This contradicts our claim that we could make a finite list of all of the prime numbers, and thereby proves that there are infinitely many primes, as claimed. \square