CCS Problem-Solving II | Professor: Padraic Bartlett

Project 3: Sequential Dynamical Systems

Weeks 1-10 | *UCSB 2015*

Mathematicians, given a process, often want to study how it **changes** or **evolves** over time. A relatively new[1] way to model such "changing systems" is with the concept of a **sequential dynamical system** (SDS, in shorthand.) Roughly speaking, a SDS consists of the following objects:

1. A **base graph**, which is a graph G — that is, some collection of points V, which we call vertices, and connections between points E, which we call edges. We think of this graph G as modeling the object we're studying; it could represent individual computers with edges corresponding to broadband lines, or cities connected by roads, or people in a social network connected by friendships.

2. A collection of **vertex states** K; these correspond to the possible "states" (like "online" or "offline" for computers, or "sick" versus "healthy" for people) that the vertices in our graph can have.

3. A collection of **vertex update** functions $\{f_v\}_{v \in V}$, that take in the state of a vertex and the states of the neighbors of that vertex, and outputs an updated vertex state. (That is, if all of your friends are sick, perhaps you will become sick as well!)

4. An **update order**, that consists of an ordering in which our vertices get updated. (For example, if you're thinking of modeling a four-way stop on a road, you don't want to have all four cars at that stop move at once! Instead, you want to evaluate them in some order.)

Given a SDS, you can use it to "simulate" your system by repeatedly updating your vertices as determined by the update order! This lets you determine how from some starting state (say, in some preset hundred-person friend network, two people are sick) what the system will look like after some number of steps forward in time.

On one hand, sequential dynamical systems are objects that are widely used to model various phenomena in the real world — from transit systems to neural networks, there are many objects that model themselves as SDS's. On the other, much of the theory behind sequential dynamical systems is still being developed, as they are a young field of study!

In this project, we will explore open problems related to sequential dynamical systems. The nature of the open problems discussed will depend on the student group's interests, backgrounds, and coding experience.

[1] The first papers to use the term "sequential dynamical systems" were published in 1999-2000.