CCS Problem-Solving I	Professor: Padraic Bartlett

Homework 14: Geometry

Due Tuesday, Week 8, at the start of class.	*UCSB 2014*

Solve **two** of the following **five** problems. (These are a bit trickier than normal, hence the lower number of problems this week.) As always, prove your claims/have fun!

0. Solve any un-signed-up-for problems from HW#13!

1. Suppose that you have been teleported back in time/space to ancient Rome. In accordance with good ancient-Roman stereotypes, you have been immediately thrown into a large gladiatorial arena, which is in the shape of a very large regular n-sided polygon.

   You are currently sitting at the center of this polygon. Meanwhile, at each vertex of your polygon there is a lion. The lions are chained so that they can only walk along the boundary of your polygon, but otherwise are unrestrained. Suppose that both you and the lions can move at up to 9m/s, can pivot and change directions instantly, are arbitrarily brilliant, and know no fear. Also, being a mathematician, feel free to assume that both you and the lions are points (i.e. neither of you have area.)

   For what values of n can you guarantee escape[1] from the n-sided polygon?

2. Same as problem 1, but you are now in the center of a perfectly circular arena, with lions posted at every point on the boundary of your circle with rational y-coordinate[2].

   Can you escape this arena?

3. Show that there is no $x \in \mathbb{R}$ such that the equation

   $$\sin(\cos(x)) = \cos(\sin(x))$$

   holds.

4. Suppose that there are two squares with side length .95 drawn inside of a disk of radius 1. Show that these two squares must overlap.

5. Can you find four points $\vec{a}, \vec{b}, \vec{c}, \vec{d} \in \mathbb{R}^2$, so that the six distances between these four points are all odd integers?

[1] "Escape," in this context, is any strategy that lets you leave the polygon in such a way that when you intersect the boundary of the polygon, there isn't a lion occupying the point that you are at. In general, lions occupying the points that you are at is not a good life strategy.

[2] Why yes, this *does* mean that you are surrounded by a countably infinite number of lions.