CCS Discrete II	Professor: Padraic Bartlett

Homework 1: Error-Correcting Codes

Due Friday, Week 1 — *UCSB 2015*

Do **one** of the **three** problems below! Prove all of your claims.

1. Historically, one of the first codes developed was the Hamming $[7,4]$ code. It works like this: take any string of four bits (i.e. any string of four 0's and 1's.) Turn this into a string of seven bits in the following way:

   - Place the bits of the original message, in order, in the slots $3,5,6,7$.
   - In slot 1, put the parity[1] of the sum of the bits in slots $3,5,7$.
   - In slot 2, put the parity of the sum of the bits in slots $3,6,7$.
   - In slot 4, put the parity of the sum of the bits in slots $5,6,7$.

   For example, to encode the message 1010, we would first place

   $$\_\_1\_010;$$

   then, because $1+0+0=1, 1+1+0=0, 0+1+0=1$, we would fill in the remaining slots to get

   $$1011010.$$

   This is a 2-ary code of length 7. Find its information rate and its minimum distance.

2. Create a 4-ary code of length 4 and distance 3, that contains 16 elements.

3. Find the largest 2-ary (i.e. binary) code of length 10 and distance 4 that you can come up with.

   Note: your score for problem 3 here is (# elements in your code)/(maximum number of elements in codes discovered by your classmates), provided that you show your work/justify your claims! I think the maximum is 40, and that we discovered this in 1980. Coding skills may be useful here.

[1] The parity of a number n is just $n \mod 2$. In other words, it is 1 if n is odd, and 0 if n is even.