

Homework 3: Current and Random Walks

*Due Friday, Week 2**UCSB 2015*

Do **one** of the following **three** problems!

1. Consider a graph G corresponding to a tiling of \mathbb{R}^2 with equilateral triangles with side length 1. Pick some vertex A to denote a starting location, and suppose we model a random walk on this graph starting from A .

What is p_{esc} ? In particular, is it 0 or nonzero?

2. Prove Rayleigh's Monotonicity Theorem:

Theorem 1. *If any of the individual resistances in a circuit increase, then the overall effective resistance of the circuit can only increase or stay constant; conversely, if any of the individual resistances in a circuit decrease, the overall effective resistance of the circuit can only decrease or stay constant.*

In specific, cutting wires (setting certain resistances to infinity) only increases the effective resistance, while fusing vertices together (setting certain resistances to 0) only decreases the effective resistance.

3. Suppose that V is a vector space with a subfield W , such that V and W are both n -dimensional. Prove that $V = W$.