

## Homework 17: Toroidal Graphs

*Due Friday, week 10**UCSB 2015*

Pick **one** of the problems in this set to solve! Solutions need justification and proof to receive full credit: i.e. it is not enough to simply draw the answer.

Also, in class I mentioned that I would have a problem describing the torus here. I decided that this was something better done in class, so look for this there!

1. Let  $G$  be a planar  $n$ -vertex graph with girth  $k$  (i.e. a graph that contains a  $k$ -cycle as a subgraph, but no smaller cycles as subgraphs.)
  - (a) Prove that  $G$  has at most  $(n - 2)\frac{k}{k-2}$  edges.
  - (b) Explain why this means the Petersen graph is nonplanar.
2. (a) Show that if  $G$  is a planar graph on 11 vertices, then the complement of  $G$  is nonplanar.
  - (b) Find a planar graph  $G$  on 8 vertices such that its complement is planar.