CCS Discrete III | Professor: Padraic Bartlett

Homework 10: Algebra and Graphs

Due Friday, Week 6 | *UCSB 2015*

Do **three** of the following **five** problems! Have fun!

1. Recall, from last quarter, the following definitions:

   Definition. Given two graphs G_1, G_2 with vertex sets V_1, V_2 and edge sets E_1, E_2, we say that a function $f : V_1 \to V_2$ is an **isomorphism** if the following two properties hold:

   - f is a bijection.
   - (x, y) is an edge in E_1 if and only if $(f(x), f(y))$ is an edge in E_2.

   An **automorphism** on a graph G is an isomorphism from that graph to itself.

   Using this definition, we say that a graph G is **vertex-transitive** if given any two vertices v_1, v_2 of G, there is an automorphism f on G such that $f(v_1) = v_2$. In essence, vertex-transitive graphs have a lot of symmetry: up to the labeling, we cannot distinguish any two vertices.

   Prove that any Cayley graph is a vertex-transitive graph.

2. Prove or disprove: there is a group A such that the Cayley graph G_A of A is (after interpreting G_A as an undirected[1] graph) is the Petersen graph.

3. Prove or disprove: there is a group A such that the Cayley graph G_A of A, when interpreted as an undirected graph, is a dodecahedron.

4. For any n, find a group G with generating set S such that its Cayley graph (again, interpreted as an undirected graph) is a K_n.

5. Let Q_n denote the graph corresponding to the n-dimensional unit cube. Find a group G with generating set S such that its Cayley graph (again, interpreted as an undirected graph) is Q_n.

[1]We do this as follows: take each of the directed graphs above and turn them into undirected graphs G_1', G_2' by "forgetting" the orientations: that is, create an edge $\{x, y\}$ in G' if and only if either $(x, y), (y, x)$ or both exist in G. Notice that the resulting graph is not a multigraph, as we only connect any x, y at most once.