CCS Discrete Math I	Professor: Padraic Bartlett

Homework 4: More Counting

Due Friday, Week 2 — *UCSB 2014*

Do **one** of the **three** problems below! Prove all of your claims. If you've seen some of these problems before, try the ones you have not seen first.

1. (a) In class, we showed that if A, B are a pair of dice with associated polynomials $A(x), B(x)$, and $D(x)$ is the polynomial associated to rolling A and B and summing their result, we have

   $$A(x) \cdot B(x) = D(x).$$

   Mimicking this result, show that if A, B, C are three dice with associated polynomials $A(x), B(x), C(x)$, then

   $$A(x) \cdot B(x) \cdot C(x) = D(x),$$

   where $D(x)$ is the polynomial associated to rolling A, B, C and summing the result.

   (b) Using this result, decide whether or not there exists a triple A, B, C of non-standard 6-sided dice such that when they are rolled and summed, they are indistinguishable from rolling and summing three standard 6-sided dice.

2. (a) Show that the polynomial

   $$p(x) = 1 + x + x^2 + x^3 + x^4$$

   is irreducible over the integer polynomials: in other words, it cannot be factored into two polynomials of strictly lower degree with integer coefficients.

   (b) Prove that there are no pairs of nonstandard 5-dice that when rolled and summed, are indistinguishable from a pair of standard 5-dice.

3. Consider the following sequence of "faux-bonacci" numbers $\{\phi_n\}_{n=0}^{\infty}$:

   $$\phi_0 = 0, \phi_1 = 4, \phi_{n+1} = \phi_n + \phi_{n-1}.$$

   Find a closed form for these numbers using generating functions, in a similar method to how we derived a closed form for the Fibonacci numbers in lecture today!