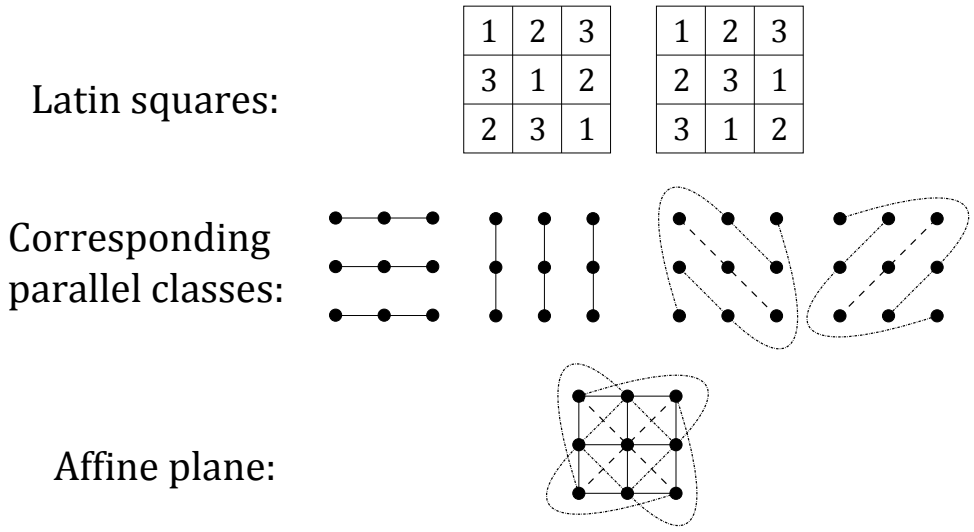
Math/CS 103 Professor: Padraic Bartlett

Handout 9: Affine Planes and MOLS

Week 5 *UCSB 2014*

Pick **two** of the **four** problems below, and solve them!

1. Prove that if p is a prime number, then there is an affine plane of order p.
   Hint: use the following connection between sets of mutually orthogonal Latin squares and affine planes, illustrated in the case where $p = 3$:



2. Take any affine plane A. Divide A's lines into $n + 1$ parallel classes $C_1, \ldots C_{n+1}$. For each class C_i, add a point ∞_i to our plane, and have every line of C_i go through ∞_i. Finally, add a line consisting of all of the points $\infty_1, \ldots \infty_{n+1}$.

   Prove that creates a projective plane.

3. Take any projective plane P. Pick a line in P, and delete that line along with all of the points on that line. Prove that the resulting set of points and lines is an affine plane.

4. Explicitly check that problems 2 and 3 work for the following specific examples:

   (a) Take the affine plane of order 3. By performing the steps of problem 2, turn it into a projective plane P

   (b) Now take the projective plane P above, and delete any line from P. (In particular, delete a line that is not the added line at infinity, because deleting that line is boring.) Show that the result is still the affine plane of order 3.