Math/CS 103 Professor: Padraic Bartlett

Handout 6: Finite Fields and Latin Squares

Week 3 *UCSB 2014*

This is due **Friday, Jan. 31.**

1. Attached to this problem set is a set of sixteen playing cards; each card has a face A,K,Q,J and a suit  , so that each pair of face and suit occurs exactly once.

   Come up with a way of arranging these sixteen cards in a 4×4 grid, so that no suit **or face** is repeated in any row or column!

2. We say that a pair of $n \times n$ Latin squares L, M are **mutually orthogonal** if the following happens: form a $n \times n$ array (L, M) by putting the ordered pair (l_{ij}, m_{ij}) in entry (i, j) of our array. If none of these ordered pairs are repeated in our entire array, we say that L and M are mutually orthogonal!

   For example, the following pair of 3×3 Latin squares are mutually orthogonal:

   $$L = \begin{array}{|c|c|c|}\hline 1&2&3\\\hline 2&3&1\\\hline 3&1&2\\\hline\end{array}, \quad L = \begin{array}{|c|c|c|}\hline 1&2&3\\\hline 3&1&2\\\hline 2&3&1\\\hline\end{array}, \quad (L,M) = \begin{array}{|c|c|c|}\hline (1,1)&(2,2)&(3,3)\\\hline (2,3)&(3,1)&(1,2)\\\hline (3,2)&(1,3)&(2,1)\\\hline\end{array}.$$

   This is because in (L, M), there are no repeated pairs of symbols.

   Find a pair of $4x4$ mutually orthogonal Latin squares.

3. For any odd prime p, find a pair of mutually orthogonal Latin squares. (Hint: look at arithmetic tables for finite fields of order p!)