Math/CS 103 | Professor: Padraic Bartlett

Homework 19: Latin Squares, Nets, and Integration

Due Friday, week 11 | *UCSB 2014*

Homework Problems.

First, recall the following definitions:

Definition. A s-**dimensional elementary interval in base** b is a subset in $\mathbb{R}^s$ of the form

$$\left[\frac{a_1}{b^{d_1}}, \frac{a_1+1}{b^{d_1}}\right) \times \left[\frac{a_2}{b^{d_2}}, \frac{a_2+1}{b^{d_2}}\right) \times \ldots \times \left[\frac{a_s}{b^{d_s}}, \frac{a_s+1}{b^{d_s}}\right)$$

for some collection of constants $a_1, \ldots a_s, d_1, \ldots d_s$. Note that all of our intervals are closed on the left and open on the right.

For example, the intervals $[1/3, 2/3), [13/9, 14/9)$, and $[2, 3)$ are all one-dimensional elementary intervals in base 3, while $[0, 2)$ is not such an elementary interval (we can't go from a to $a + 2$) and neither is $[5/3, 6/7)$ (the denominator changes) or $[1/3, 2/3]$ (the right side is closed.)

Similarly, $[1/4, 2/4) \times [5/2, 6/2), [3/8, 4/8) \times [17/16, 18/16)$, and $[1, 2) \times [0, 1/2)$ are all two-dimensional elementary intervals in base 2.

Definition. A (t, m, s)-net in base b is a collection of b^m points in $[0, 1)^s$, such that every elementary interval E of volume $1/b^{m-t}$ contains exactly b^t points.

Pick **two** of the following **four** problems to solve!

1. Consider the integral
$$\int_{[0,1]^4} wxyz \; dwdxdydz.$$
   (a) Directly calculate this integral.
   (b) Approximate this integral by choosing 9 points at random in $[0, 1]^4$, plugging them into the function $wxyz$, and averaging.
   (c) Approximate this integral by using the $(0, 2, 4)$ net we constructed in class.

2. Do the same task as in problem 1, but with a function of your own construction and net of your choice! The only restrictions are that you pick a function f that takes in at least three variables, and is not constant in any of these variables (i.e. $f(w, x, y, z) = 0$ is boring, don't pick it.)

3. Prove the claim we made in class: if we have a set of $s - 2$ mutually orthogonal Latin squares of order b, then we can create a $(0, 2, s)$-net in base b. In particular, explain why our construction actually creates something that **is** a net.

4. Find three nets, such that (1) each net contains at least 5 points, and (2) are **not** of the form $(0, 2, s)$.