## Homework 3: Presentations from Week 1

## Homework Problems.

Pick two of the following three problems to solve!

1. (Nick's problem.)
(a) Explain why the grid below cannot be a starting grid for method Nick showed for completing Latin rectangles:

| $1^{\prime}$ | $2^{\prime}$ | $3^{\prime}$ | $4^{\prime}$ |
| :---: | :---: | :---: | :---: |
| $X_{1}$ | $X_{1}$ | 1 | 1 |
| 2 | $X_{2}$ | 2 | $X_{2}$ |
| $X_{3}$ | $X_{3}$ | $X_{3}$ | 3 |
| 4 | 4 | $X_{4}$ | $X_{4}$ |

(b) Complete the following Latin rectangle using this method:

| 2 | 4 | 5 | 1 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| 3 | 5 | 4 | 2 | 1 |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

Use the following three tables:

| $1^{\prime}$ | $2^{\prime}$ | $3^{\prime}$ | $4^{\prime}$ |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |



(c) What is the run-time of this algorithm for an $n x n$ Latin square with $r$ filled rows?
2. In class, we defined the game Sim, which was a 2-player game on a set of six vertices. We proved that this game never ends in a draw. Suppose that you are playing this game, and you get to choose whether you go first or second. Which should you pick if you want to win? Why? Can either player guarantee that they will always win?
3. (a) Find $R(3,4)$ : i.e. find the smallest value of $n$ such that any two-coloring of $K_{n}$ gives you either a red triangle or a blue $K_{4}$.
(b) Suppose you are playing a "handicapped" version of Sim, where the red player is trying to avoid making a red triangle and the blue player is trying to avoid making a blue $K_{4}$. In part (a), you found a value $n$ such that games of this handicapped version on $n$ vertices are guaranteed to not end in a draw. Suppose you play this game on $n-1$ vertices. Can the red player force a victory? Or can the blue player force a draw?

