Math/CCS 103 — Professor: Padraic Bartlett

Homework 17: Presentations (Nick and Landon)

Due Friday, week 9 — *UCSB 2014*

Do **two** of the **four** problems below!

1. (Landon) The Avengers and our Caped Crusader want to lock away that evil villain Loki once and for all. What better way to do this than to lock him up in a special safe that Tony Stark created that has a Latin square as its password/combination?

   However, the genius Tony Stark is having some trouble making the combination and determining which partial Latin square keys to give to who, because Tony never had Paddy for a teacher. And Batman is o doing bat stu so he cant solve the problem for Tony.

   Help save the day by coming up with a minimal critical set of any size you like and dividing the cells into dierent partial keys such that:

   - Tony Stark (Team Alpha) only needs to consult with any one other person to unlock the safe.
   - Batman, Robin, Thor, Black Widow, and Captain America (Team Beta) each need to consult one of
     - just Tony Stark,
     - two other members from Team Beta, or
     - Team Gamma and one person from Team Beta,

     if they want to open the safe.
   - The Hulk (Team Gamma) needs to consult with either Tony Stark or at least four members from Team Beta to unlock the safe — the Hulk can sometimes make impulsive decisions, so we want to make sure that the others know what he is up to before he does anything that may cause problems.

2. (Landon) Construct a critical set for a 4×4 Latin square that is not minimal, and show why it is not minimal. Remember that removing a single element from a critical set makes it completeteable into more than one Latin square.

3. (Landon) Can you create a minimal critical set for a 7×7 Latin square that is not the standard critical set? (Hint: Look at the way I began constructing them in the lecture.)

4. (Nick) Recall that the formula for curvature is

$$K = \left| \left| \frac{\frac{d^2y}{dx^2}}{[1 + (\frac{dy}{dx})^2]^{3/2}} \right| \right.$$

   (a) Suppose that we are looking at the curve $y = -\ln(|\cos(x)|)$. Show that the curvature of this curve is $|\cos(x)|$.

(b) Ideally, we would hope that the the curvature of a curve would not be a “local” property: i.e. we might hope that if we examine the quantity K above at several points on our curve, it would be the same everywhere. For an arbitrary curve $y = f(x)$, find the curvature equations (as a function of x) of $y = f(x)$ and $y = f(x-k)+h$. Are these the same quantities? (If so, then our hope above is validated! If not, then we are sad.)

(c) Suppose we measure the curvature of a curve $y = f(x)$ at a critical point (i.e. a point x such that $f'(x) = 0$.) Does anything interesting happen here?