Math/CCS 103 Professor: Padraic Bartlett

Homework 12: Presentations (Zihan, Kayla and Declan)

Due Friday, week 6 *UCSB 2014*

Do **two** of the **five** problems below!

1. (a) (Zihan) Calculate the value of the Legendre symbol $(\frac{-32}{97})$.

   (b) (Zihan) Is there any solution to the congruence $x^2 \equiv 5 \bmod 227$?

   Hints: Here are some special formulas for the Legendre symbol$(\frac{a}{p})$ for small values:

   - $(\frac{-1}{p}) = (-1)^{\frac{p-1}{2}}$.
   - $(\frac{2}{p}) = (-1)^{\frac{p^2-1}{8}}$.
   - For an odd number prime $p \neq 3$, $(\frac{3}{p}) = (-1)^{\frac{p+1}{6}}$.

2. (Declan) What is the minimum number of guards sufficient to see every point of the interior of an n-vertex simple polygon if those guards are half-guards? Which means they are fixed 180° field of vision.

3. (Declan) What is the minimum number of guards sufficient to see every point of the interior of an n-vertex simple polygon if each guard is also visible to at least one other guard? What about they are in an orthogonal art gallery?

4. (Declan) Can you prove any claim about the minimum number of guards sufficient to see every point of the interior of an orthogonal art gallery with h-holes and n-vertex?

5. (Kayla) Music! Specifically: consider the following table, which gives us a way to turn musical notes into a group:

$+$	C	$C^\sharp$	D	$D^\sharp$	E	F	$F^\sharp$	G	$G^\sharp$	A	$A^\sharp$	B
C	C	$C^\sharp$	D	$D^\sharp$	E	F	$F^\sharp$	G	$G^\sharp$	A	$A^\sharp$	B
$C^\sharp$	$C^\sharp$	D	$D^\sharp$	E	F	$F^\sharp$	G	$G^\sharp$	A	$A^\sharp$	B	C
D	D	$D^\sharp$	E	F	$F^\sharp$	G	$G^\sharp$	A	$A^\sharp$	B	C	$C^\sharp$
$D^\sharp$	$D^\sharp$	E	F	$F^\sharp$	G	$G^\sharp$	A	$A^\sharp$	B	C	$C^\sharp$	D
E	E	F	$F^\sharp$	G	$G^\sharp$	A	$A^\sharp$	B	C	$C^\sharp$	D	$D^\sharp$
F	F	$F^\sharp$	G	$G^\sharp$	A	$A^\sharp$	B	C	$C^\sharp$	D	$D^\sharp$	E
$F^\sharp$	$F^\sharp$	G	$G^\sharp$	A	$A^\sharp$	B	C	$C^\sharp$	D	$D^\sharp$	E	F
G	G	$G^\sharp$	A	$A^\sharp$	B	C	$C^\sharp$	D	$D^\sharp$	E	F	$F^\sharp$
$G^\sharp$	$G^\sharp$	A	$A^\sharp$	B	C	$C^\sharp$	D	$D^\sharp$	E	F	$F^\sharp$	G
A	A	$A^\sharp$	B	C	$C^\sharp$	D	$D^\sharp$	E	F	$F^\sharp$	G	$G^\sharp$
$A^\sharp$	$A^\sharp$	B	C	$C^\sharp$	D	$D^\sharp$	E	F	$F^\sharp$	G	$G^\sharp$	A
B	B	C	$C^\sharp$	D	$D^\sharp$	E	F	$F^\sharp$	G	$G^\sharp$	A	$A^\sharp$

   (a) Explain why this is a group.

   (b) Find all of the possible subgroups of this group.

   (c) Is there a nice musical interpretation for any of these subgroups? How about for their cosets?