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Lecture :Combinatorial Games

Lecture Notes *UCSB 2014*

What are Combinatorial Games?:

Combinatorial games are a specific type of game within game theory. They are different than typical game theory games in the fact that all must meet the following requirements:

1. There are two players who alternate moves throughout the game.

2. There are finite positions which follow rules regarding moves that players can make from any given position to its given options.

3. All games will resolve to some ending condition, i.e. there is no possibility of draws or ties. All games will have a winner.

4. Both players have perfect information of the possible moves for both themselves and their opponent.

5. There are no chance devices that determines the outcome of the game, i.e. no dice or shuffled cards are used.

With these conditions, we are able to determine what games are and aren't combinatorial games. Many card and dice based games such as poker, blackjack, and Yahtzee all have chance devices, thus not satisfying the fifth condition. Many of these games also do not have perfect information and primarily rely on your opponent not knowing your possible moves (or in this case cards). Games like Tic-Tac-Toe and chess have the potential to end in ties or draws and not reaching a decisive winner, breaking condition 3. Try thinking about some of your favorite games to play and determine if they are combinatorial or not!

Impartial and Partizan

Within combinatorial games, there are two different classifications of a game: impartial and partizan. An impartial game is one where the same moves exist for both players where as in a partizan game each player controls their own set of moves. An example of an impartial game is SIM, the game we learned about in class. Since the two players are given an empty K_6 graph, they both have the same possible edges (moves) to select from, making it impartial. While chess is not a perfect combinatorial game, as I said before, it is a great example of a partizan game. Both players make the same possible moves just with their own separate pieces. Depending on the classification of the game, ones strategy for winning will be changed drastically. This makes it important to establish exactly what kind of combinatorial game you are playing before trying to devise a winning strategy.

Game Trees

Since combinatorial games have finite positions that are known to both players, the options and course of a game can be mapped by a game tree. A game tree is comprised of nodes and edges, representing the positions and potential moves at any point in the game. An example of a game tree for Tic-Tac-Toe can be seen at:

http://www.emn.fr/z-info/pdavid/Enseignement/IA/poly-ia/jeux/arbre-jeu-tic-tac-toe.html

As you can see from the example, the more options within a game, the more complex the game tree. However, these representations can be very helpful in determining a winning strategy by "playing out" the game.