Parrondo's paradox

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Question

Is it possible to set up two losing gambling games such that, when they are played alternatively, they become winning?

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Is there a winning strategy?

Yes! If we play the games alternatively, starting with Game B, followed by A, then by B, and so on (BABABA...), we will steadily earn $2 for every two games.

Parrondo's paradox

Parrondo's paradox is a paradox in game theory. It was discovered by Juan Parrondo in 1996. Its description is:

There exist pairs of games, each with a higher probability of losing than winning, for which it is possible to construct a winning strategy by playing the games alternately.

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2. In Game A, we toss a biased coin, Coin 1, with probability of winning $P_1 = (1/2) - \epsilon$. If $\epsilon > 0$, this is clearly a losing game in the long run.

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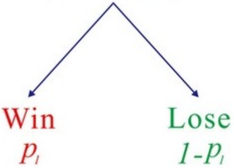
2. In Game A, we toss a biased coin, Coin 1, with probability of winning $P_1 = (1/2) - \epsilon$. If $\epsilon > 0$, this is clearly a losing game in the long run.
3. In Game B, we first determine if our capital is a multiple of some integer M. If it is, we toss a biased coin, Coin 2, with probability of winning $P_2 = (1/10) - \epsilon$. If it is not, we toss another biased coin, Coin 3, with probability of winning $P_3 = (3/4) - \epsilon$.

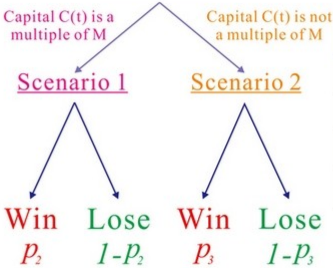
Capital-dependent Parrondo's paradox

Game A



Game A	
p_1 (Win)	$1-p_1$ (Lose)
0.5 -ε	0.5 +ε

Game B



Game B			
Is capital C(t) a multiple of M?			
Scenario 1 (Yes)		Scenario 2 (No)	
p_2 (Win)	$1-p_2$ (Lose)	p_3 (Win)	$1-p_3$ (Lose)
0.1 -ε	0.9 +ε	0.75 -ε	0.25 +ε

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However, when these two losing games are played in some alternating sequence - e.g. two games of A followed by two games of B (AABBAABB...), the combination of the two games is, paradoxically, a winning game.

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However, when these two losing games are played in some alternating sequence - e.g. two games of A followed by two games of B (AABBAABB...), the combination of the two games is, paradoxically, a winning game.

Not all alternating sequences of A and B result in winning games. For example, one game of A followed by one game of B (ABABAB...) is a losing game, while one game of A followed by two games of B (ABBABB...) is a winning game.

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While Game B is a losing game under the probability distribution that results for C_t modulo M when it is played individually, it can be a winning game under other distributions, as there is at least one state in which its expectation is positive.

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As the distribution of outcomes of Game B depend on the player's capital, the two games **cannot** be independent. If they were, playing them in any sequence would lose as well.

M serves to induce a dependence between Games A and B, so that a player is more likely to enter states in which Game B has a positive expectation, allowing it to overcome the losses from Game A.

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With this understanding, the paradox resolves itself: The individual games are losing only under a distribution that differs from that which is actually encountered when playing the compound game.

Is Parrondo's paradox really a "paradox"

"Parrondo's paradox" is just a name. Most of these named paradoxes they are all really apparent paradoxes. People drop the word "apparent" in these cases as it is a mouthful, and it is obvious anyway. So no one claims these are paradoxes in the strict sense. In the wide sense, a paradox is simply something that is counterintuitive. Parrondo's games certainly are counterintuitive - at least until you have intensively studied them for a few months.

- Derek Abbott

Application

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Parrondo's games are of little practical use such as for investing in stock markets[10] as the original games require the payoff from at least one of the interacting games to depend on the player's capital.

Sources

http://en.wikipedia.org/wiki/Parrondo's_paradox
http://www.nature.com/srep/2014/140228/srep04244/full/srep04244.html

Thank you

Thank you all for listening to my presentation.

Questions?