



Homework 3: Exploring the Seas

*Due 10/4/13, at the start of class**UCSB 2013*

Choose **three** of the **five** problems below to complete by next class. Be ready and able to present your solutions if you have them, or your questions if you don't solve the problems!

1. More submarines! Specifically, you're now in charge of creating submarines. Your task is simple: create a submarine that can get to any point in the ocean!

In particular, you want to create a number of these submarines, each with various restrictions. For each restriction, either create such a submarine if it is possible, or prove that it is impossible if you cannot create such a submarine.

- (a) Can you create a submarine with three engines that can make it to any point in the ocean?
 - (b) Can you create a submarine that can make it to any point in the ocean, such that for each engine (a, b, c) , we have $a > b > c > 0$?
 - (c) Can you create a submarine that can make it to any point in the ocean, such that each engine is of the form $(a, a + 1, a + 2)$?
2. Same thing as problem 1, just with new conditions:
 - (a) Can you create a submarine that can make it to any point in the ocean, such that each engine (a, b, c) has the property $a + b + c = 1$?
 - (b) Can you create a submarine that can make it to any point in the ocean, such that each engine (a, b, c) has the property $a + b + c = 0$?
 - (c) Can you create a submarine that can make it to any point in the ocean, such that each engine (a, b, c) has the property $a + b = 0$?
 3. Consider submarines restricted to the surface of the ocean: i.e. submarines such that the z -component of their engines is always 0.
 - (a) Show that if such a restricted submarine is capable of going to any point on the surface of the ocean, then it must have at least two engines.
 - (b) Take any restricted submarine with two engines that can travel to any point on the surface of the ocean. Add any engine with a nonzero z -component. Show that this submarine can go to any point in the ocean.
 4. Show that if you want to create a submarine to explore all of the ocean, you need at least three engines. In other words, show that no submarine with two engines can explore every single point of the ocean.
 5. Take any pair of submarines , , with the following characteristics:
 - Both of these submarines have two engines apiece.

- None of these engines are “multiples” of each other: i.e. if (x, y, z) is an engine of a submarine, then (ax, ay, az) is not the second engine of that submarine, for any number a .
- Both of these submarines start at $(0, 0, 0)$.

Show that there is some point $(x, y, z) \neq (0, 0, 0)$ in the ocean that both of these submarines can travel to.