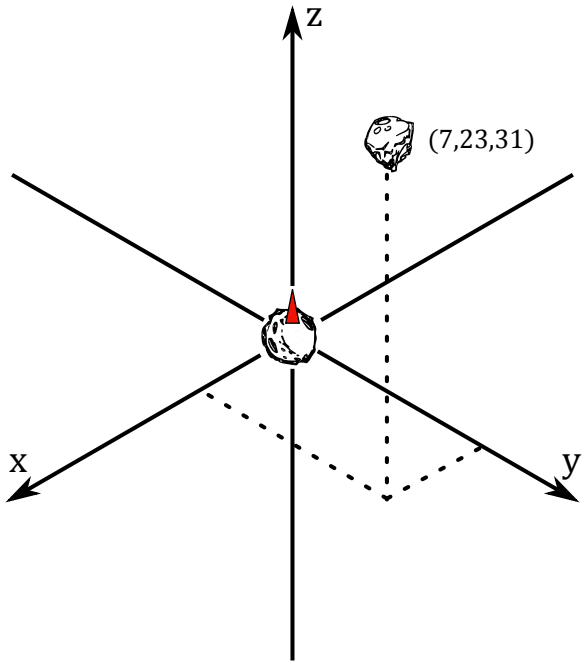
Math/CS 103 Professor: Padraic Bartlett

Homework 1: Seas and Space

Due 9/30/13, at the start of class *UCSB 2013*

Choose **three** of the **five** problems below to complete by next class. Be ready and able to present your solutions if you have them, or your questions if you don't solve the problems!

1. You have been hired by an interstellar mining company working in the Oort cloud as a navigator. Your first job is the following: to get a shuttle from our home base (located at $(0, 0, 0)$ in the drawn picture) to a nearby interesting asteroid, which at the time of launch will be at $(7, 23, 31)$.



You have your choice of four possible shuttles:

 : Shuttle 1 has three rockets attached to it. Firing one of these rockets will move our shuttle either $(+1, +1, +0)$, $(+0, +1, +1)$, or $(+0, +0, +1)$ units (depending on the rocket chosen) from its current location[1] before the shuttle comes to rest.

 : Shuttle 2 has two rockets attached to it, which from any given location can move it either $(+1, +2, +3)$ or $(+1, +0, +0)$ units.

 : Shuttle 3 has three rockets attached to it, which from any given location can move it either $(+2, +4, +1)$, $(-2, +6, +3)$,or $(+2, -1, -1)$ units.

[1] The reason that firing a given rocket doesn't cause our ship to move forever is the drag from running into particles in the Oort cloud, in case you were wondering.

: Shuttle 4 has four rockets attached to it, which from any given location can move it either $(+2, +3, +4)$, $(+1, +3, +1)$, $(+2, +1, +1)$, or $(+0, -1, +4)$ units.

Each rocket can be fired as many times as you want, but can only be fired for the full duration of their burn. For example, a $(+1, +2, +2)$ rocket could be fired once to move a shuttle $(+1, +2, +2)$ units, or twice to move the shuttle $(+2, +4, +4)$ units; however, it could not "half-fire" itself to move $(+.5, +1, +1)$ units.

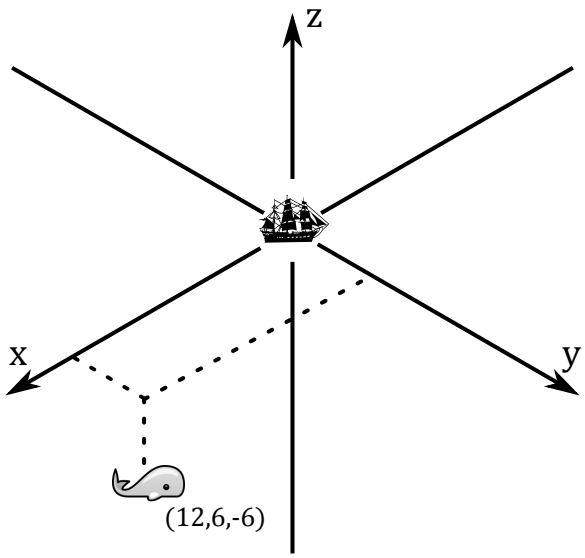
For each of these rockets, determine whether or not said shuttle can make it to the nearby asteroid. If the rocket can make it to the asteroid, provide a set of instructions that describe how to do this. If the rocket cannot make it to the asteroid, please explain why not.

2. Suppose that the rockets you're working with in problem 1 have been **upgraded**: now, instead of requiring us to fire them for the full duration of their burn, we can fire them for **fractional** amounts of their burn. For example, a $(+1, +2, +2)$ rocket can now be fired for $\frac{1}{4}$ of its duration to move a shuttle $\left(+\frac{1}{4}, +\frac{1}{2}, +\frac{1}{2}\right)$ units.

   How does this change your answers to problem 1? Are there any shuttles that can now make it to the asteroid that could not before? Are there any shuttles that still cannot make it?

3. A swarm of small iron-rich meteroites have appeared close to our launching pad. In particular, there are meteorites at every single point of the form (x, y, z), where x, y, z are integer values ranging from 1 to 4. Consider the upgraded shuttles from question 2. For each of these shuttles, determine whether or not they can reach all of the meteorites in the swarm. If they cannot, identify the total number of meteorites that can be reached.

4. Based off of your success with the mining project, you've also been hired as a deep-sea navigator. Your tasks here are similar: starting from a ship moored at $(0, 0, 0)$, you want to pilot a submarine to observe a whale pod at $(12, 6, -6)$.



However, this time you're working with submarines, which have a few advantages over space shuttles:

- Like the upgraded shuttles in question 2, we can fire up any given engine for a submarine for fractional amounts of time.
- However, unlike the upgraded shuttles, we can also **reverse** our engines to travel **backwards**! For example, a $(+4, +4, -2)$ engine could be ran backwards for $-\frac{1}{2}$ cycles, which would translate our submarine by $(-2, -2, +1)$ units from its current location.

Consider the following triple of submarines:

 : Submarine 1 has three engines attached to it, which (when ran for a cycle from any given location) can move it either $(1, 0, 1)$, $(0, 1, 1)$, or $(1, 1, 0)$ units from its current location.

 : Submarine 2 has two engines attached to it, which from any given location can move it either $(4, 2, 0)$ or $(6, 6, -1)$ units.

 : Submarine 3 has three engines attached to it, which from any given location can move it either $(3, 0, 1)$, $(1, 0, 3)$, or $(1, 1, 1)$ units in any given direction.

Which of these three submarines can make it to the whales? Create a set of instructions for the submarines that can make it to the whales, and write an explanation for any submarines that cannot.

5. Suppose that in question 4, a violent storm has created a strong current in the area. Our boat is anchored, so it won't move, but any submarine that we release in the ocean will be pushed by $(-2, -2, -1)$ units for every cycle of time. So a submarine released from our ship will be at $(-2, -2, -1)$ after one cycle of time if it never fires its engines. However, if it has a $(+4, +2, -1)$ engine and runs it for a cycle when it is released, it will instead be at $(+2, 0, -2)$ after one cycle of time.

   Which of the three submarines can make it to the whales now? Again, create a set of instructions for the submarines that can make it to the whales, and write an explanation for any submarines that cannot.