Math 7h — Professor: Padraic Bartlett

# Homework 7: Latin and Magic Squares

*Due Tuesday, week 8, at the start of class* — *UCSB 2014*

Try one of the following three problems (or come up with something of your own!)

1. (a) Is there a diagonal $6 \times 6$ Latin square? Find one, or prove no such thing can exist.

   (b) Is there a $6 \times 6$ magic square? Find one, or prove no such thing can exist.

2. Prove the claim we made in class:

   **Proposition.** Take any $n \times n$ diagonal Latin square $L$ on the symbols $\{1, \ldots n\}$ . Form the array $M$ as follows: in entry $(i, j)$ of $M$, place the sum $L(i, j) + n \cdot L(j, i) - n$. I.e. to fill the square $(i, j)$, add whatever symbol is in $L(i, j)$ to $n \cdot L(j, i)$, and subtract $n$.

   Then $M$ is a magic square!

3. A **broken right diagonal**, or **wraparound right diagonal**, in a Latin square $L$ is the set of $n$ cells acquired by starting from one of the cells in our top row and repeatedly taking the cell that's one below and one to the right of this cell, wrapping around our square if we hit the last column, until we get to the last row.

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   A **broken left diagonal** is the same kind of object, except wrapping around to the left instead of the right.

   Given these definitions, a Latin square $L$ is called **pandiagonal** (alternately, **diabolic**, or **perfect**, depending on the author) if every broken diagonal contains no repeated symbols.

   For what values of $n$ can you find a pandiagonal Latin square? Find at least one $n \times n$ pandiagonal Latin square for $n > 1$.