Homework 5: Infinite Random Graphs

Try one of the following three problems (or come up with something of your own!) You may need to look up definitions to do a problem; feel free to either stop by OH, email, use Wikipedia/etc to fill in gaps in your knowledge.

1. Take the Rado graph R from lecture. Delete finitely many vertices and edges from this graph, to get some new graph R'. Is R' isomorphic to R? If it is, prove your claim; if it is not, give a construction that explains why this fails.

2. Consider the following graph on the vertex set $\mathbb{N}$, where we draw an edge $\{x, y\}$ whenever the x-th bit of y's binary representation is 1, or the y-th bit of x's binary representation is 1. Show that this graph is isomorphic to the Rado graph.

3. At the end of the lecture notes, we created a map φ, and claimed it was an isomorphism. Prove that φ actually is an isomorphism!