Math 7h	Professor: Padraic Bartlett

Homework 5: P vs. NP

Due 11/5/13, at the start of class	*UCSB 2013*

Instructions: Five classes of problems are listed below. Pick **one** class of problem, and attempt to do the following:

- Find an algorithm that solves the problem. Check the runtime of your algorithm. (It will likely be huge.)
- Show that your problem is in NP: i.e. find an algorithm that will take in an instance of your problem and a "proof" that claims to show that instance is true, and check in polynomial time whether that solution holds.
- Then, try to improve your algorithm in such a way that your problem is in P. (This may be difficult.)

As always, work on problems until you either spend about 90m on the questions or you solve a problem.

Homework Problems

1. Take an arbitrary $n \times n$ partial latin square P. Does it have a completion to an $n \times n$ latin square L ,in which all of its rows and columns are filled?

2. In a graph $G = (V, E)$, a **Hamiltonian cycle** is a sequence of vertices and edges $(v_1, e_{12}, v_2, e_{23}, \ldots v_n, e_{n1}$, such that

   - each vertex in V shows up in our sequence exactly once, and
   - the edges e_{ij} are all edges linking vertex v_i to vertex v_j.

   In other words, a Hamiltonian cycle is a tour that starts and stops at the same vertex, and along the way visits every other vertex exactly once.

   Given an arbitrary graph G on n vertices, does it have a Hamiltonian cycle?

3. A 3-**coloring** of a graph G is a way to assign the colors $\{1, 2, 3\}$ to the vertices of a graph in such a way that no edge has both of its endpoints colored the same color.

   Given a graph G, does it have a 3-coloring?

4. Take a graph G. We can play a solitaire game, called **pebbling**, on this graph. We define this as follows:

   - Setup: a graph G. Also, to every vertex of G, we assign some number of "pebbles," which we imagine are stacked on top of each vertex.
   - Moves: Suppose we have an edge e_{12} connecting v_1 to v_2, and another edge e_{23} connecting v_2 and v_3. Suppose further that there is a pebble on v_1 and v_2. We can then "jump" the pebble v_1 over the pebble at v_2 to v_3: i.e. we can remove one pebble from each of v_1 and v_2, and place a pebble on v_3.
   - A game is **cleared** if we can reduce it to having only one pebble on the entirety of the board.

   Given an arbitrary graph G on n vertices, and some arrangement of n pebbles on G, can this game ever be "cleared"?

5. Consider the following puzzle:

   - Take a $n \times n$ board. To set up a puzzle, place some red and blue stones on the squares in the grid, so that each square is either empty, contains a blue stone, or contains a red stone.
   - The goal of this puzzle is to remove stones so that the following properties hold:
     - Every row contains at least one stone.
     - No row contains both a red and a blue stone.

     For some initial configurations of stones, this is impossible (find one!).

     Given a game on an $n \times n$ board, can you solve this puzzle?