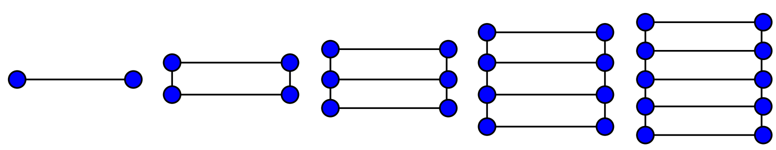
Math 7h	Professor: Padraic Bartlett
Homework 4: Ramsey Theory	
Due 10/29/13, at the start of class	*UCSB 2013*

Instructions: Choose **one** of the problems below, and work on it until either:

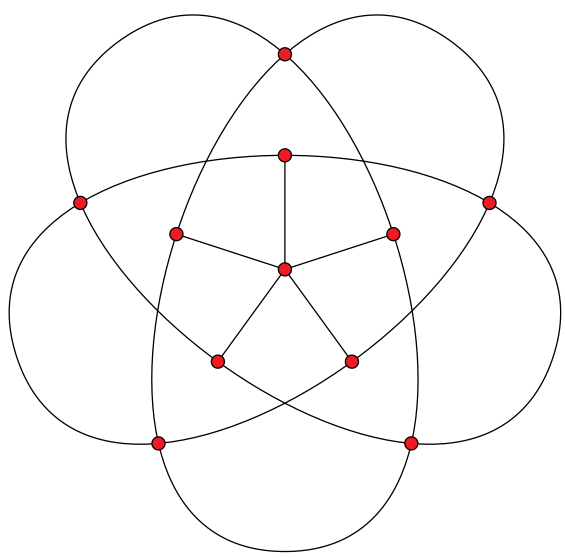
1. You solve the problem, or
2. You have spent about 90 minutes working seriously on the problem.

Homework Problems

1. A **proper k-edge coloring** of a graph $G = (V, E)$ is a way to assign k distinct colors to the edges of the graph G in such a way that no vertex is incident with two distinct edges of the same color. Find the edge-chromatic number of the following graphs:
   - K_n.
   - The ladder graphs L_n, for any n (depicted below for $n = 1, 2, 3, 4, 5$.)



   - The Petersen graph.
   - The Grötzch graph (depicted below.)



2. Find $R(3, 5)$.
3. We have shown that the Ramsey numbers have bounded growth from above. Can you find an explicit bound for the growth of the diagonal Ramsey numbers $R(n, n)$? More specifically, can you find a function $f(n)$ such that $R(n, n) \leq f(n)$? How small can you get $f(n)$ to be?
4. Find a construction that shows $R(3, t + 1) > 3t - 1$.
5. Show that every set of $B = \{b_1, \ldots b_n\}$ of n nonzero integers contains a sum-free[1] subset of size $\geq n/3$.

[1] A subset of $\mathbb{R}$ is called sum-free if adding any two elements in the subset will never give you an element of the subset.