Math 108a	Professor: Padraic Bartlett

Homework 8: More Matrices

Due Friday, 11/22/13, 1:30 pm — *UCSB 2013*

As always, problems need to show work and proofs in order to receive full credit. Have fun!

1. In the last HW, we looked at the map $T_\theta : \mathbb{R}^2 \to \mathbb{R}^2$, that rotates a vector in the plane counterclockwise by theta. Specifically, we looked at this linear map, and showed that it is the matrix
$$T_\theta = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}.$$
Take two such matrices T_θ, T_γ, and calculate the matrix
$$T_\theta \circ T_\gamma.$$
Show that it is equal to the matrix $T_{\gamma+\theta}$.

2. For a given matrix M, let M^n denote the matrix acquired by composing M with itself n times: i.e.
$$M^n = \overbrace{M \circ M \circ \ldots \circ M}^{n \text{ times}}.$$
Let $E_{\text{multiply entry } k \text{ by } \lambda}$ denote the elementary matrix that we talked about in class this Monday. Find the matrices corresponding to
$$\left(E_{\text{multiply entry } k \text{ by } \lambda}\right)^2, \left(E_{\text{multiply entry } k \text{ by } \lambda}\right)^3, \left(E_{\text{multiply entry } k \text{ by } \lambda}\right)^4.$$
In general, find a formula for
$$\left(E_{\text{multiply entry } k \text{ by } \lambda}\right)^n.$$
Explain why your answer is correct.

3. (a) Suppose that P is an $n \times n$ matrix such that $P^2 = P$. Prove that $P^n = P$, for any positive integer n.
   (b) For any n, find an $n \times n$ matrix P, not equal to the identity matrix, such that $P^2 = P$.

4. Write the matrix
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$
as a product of elementary matrices.

5. Write the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

as a product of elementary matrices.

6. How long did this set take you? (As always, asked solely for calibration purposes.)