NP and Latin Squares	Instructor: Padraic Bartlett
Homework 3: Triangulations and NP-Completeness	
Week 4	*Mathcamp 2014*

Homework Problems.

1. (a) Prove that if a graph G has each of its vertices with odd degree, then G does not admit a triangulation.
   (b) Prove that if the number of edges in G is not divisible by 3, then G does not admit a triangulation.
   (c) Find a graph G where every vertex has even degree and the number of edges is a multiple of 3, but G does not admit a triangulation.
   (d) (Open!) Find a value of ϵ such that any graph on n vertices with minimum degree $(1-\epsilon)n$ that satisfies properties (a) and (b) admits a triangulation. (Conjectured bound is 1/4 here.)

2. Find a complete graph K_n such that
   - K_n is decomposable into triangles, and
   - $n > 3$.

3. Show that if n is congruent to 1 or 3 mod 6, then K_n admits a decomposition into triangles.

4. A 4**-cycle decomposition** is basically a triangle decomposition, except with squares (i.e. 4-cycles): i.e. it is a way to break the edges of a graph into disjoint subsets, each one of which forms a 4-cycle.
   (a) Explain why if a graph has a 4-cycle decomposition, the degree of every vertex must be even and the number of edges must be a multiple of 4.
   (b) Find a graph that has every vertex of even degree and its number of edges a multiple of 4, but does not have a 4-cycle decomposition.
   (c) Find a complete graph K_n that has a 4-cycle decomposition.

5. Generalize problem 3: for any m, find a n such that K_n has a m-cycle decomposition.