Homework 2: Random and Quasirandom Graphs

Attempt all of the problems that seem interesting, and let me know if you see any typos! $(-)$ problems are maybe tedious to attempt. $(+)$ problems are harder than the others. $(++)$ problems are currently open.

1. $(-)$ Check by hand that the total number of triangles in the collection of all graphs on 4 vertices is 32.

2. Let C_5 be a 5-cycle. At what value of n does the total expected number of 5 cycles in a random graph on n vertices exceed 1?

3. Let P be the Petersen graph. At what value of n does the total expected number of copies of P in a random graph on n vertices exceed 1?

4. Create a few random graphs and check their eigenvalues (probably with Mathematica/Wolfram Alpha if you want this to not take a while). Do you see eigenvalues that correspond with what we predicted in class?

5. A graph is called **connected** if there is a path between any two vertices within the graph. Show that as $n \to \infty$, the probability that a random graph is connected goes to 1.