Many Campers Sort Piles	Instructor: Padraic Bartlett
Homework 4: Many Campers Sort Piles	
Week 4	*Mathcamp 2012*

Attempt all of the problems that seem interesting, and let me know if you see any typos! (+) problems are harder than the others. (++) problems are currently open.

1. Consider the following recursive algorithm $Factor(n)$ for finding $n!$, given a nonnegative integer input n:

   - If $n = 0$ or 1, return 1.
   - Otherwise, return $n \cdot Factor(n-1)$.

   (a) How many steps does this take to run?

   (b) How many bits are required to write $n!$ in binary, roughly speaking? (Use Stirling's approximation, which says that $n! \approx \sqrt{2\pi n} \cdot (n/e)^n$.)

   (c) Given your answer above, you might believe that the run time you calculated in (a) is far too large for such a simple task! Consider instead the following better algorithm $Factor2(n, m)$ which computes $(n!)/(n-m)!$:

   - If $m = 0$, return 1.
   - Othewise, if $m = 1$, return n.
   - Otherwise, return $Factor2(n, \lfloor m/2 \rfloor) \cdot Factor2(n - \lfloor m/2 \rfloor, \lceil m/2 \rceil)$.

   How many steps does $Factor2(n, n)$ take to calculate $n!/0!$, roughly?

2. Spaghetti sort is a sorting algorithm that uses **spaghetti**! We define it here:

   (a) Input: a list of integers; also, a box of dried spaghetti, a hand, and a table.

   (b) Given this list of integers, find the largest integer m in our list.

   (c) Define a full piece of spaghetti as m spaghetti-units. Using these units and your box of spaghetti, create a piece of spaghetti of length k for every number k in our list.

   (d) Take all of your newly-formed spaghetti-integers, put them in your hand, take them loosely in your hand and lower them to the table, so that they all stand upright, resting on the table surface. They are now sorted by height!

   (e) One by one, pick out the shortest rod of spaghetti and write it down on our list. This process orders our list.

   What is the run time of this algorithm? Include your assumptions on how much time it takes you to perform the various steps in this algorithm.

2. Sleepsort is perhaps most famous for being the only sorting algorithm discovered by 4chan. We define it here:

(a) Input: a list $\{l_1, \ldots l_n\}$ of n distinct integers. Also, someone with a stopwatch, and another person with a pen.

(b) Start your stopwatch.

(c) Every time the number of seconds is equal to a number on the list, the person with the stopwatch should shout out the number, and the person with a pen should write it down.

(d) After a number of seconds equal to the size of the largest element in your list, you've written down all of the numbers in order! Win.

2. What is the run time of this algorithm? Again, make sure to be clear about your assumptions. What problems does this algorithm run into for large lists of numbers?