Attempt all of the problems that seem interesting, and let me know if you see any typos! (+) problems are harder than the others. (++) problems are currently open.

1. Find a code with infomation density 1.

2. What is the minimum distance of the code formed by taking all words in English as your codewords?

3. Historically, one of the first codes developed was the Hamming $[7, 4]$ code. It works like this: take any string of four bits (i.e. any string of four 0's and 1's.) Turn this into a string of seven bits in the following way:

   - Place the bits of the original message, in order, in the slots $3, 5, 6, 7$.
   - In slot 1, put the parity of the sum of the bits in slots $3, 5, 7$.
   - In slot 2, put the parity of the sum of the bits in slots $3, 6, 7$.
   - In slot 4, put the parity of the sum of the bits in slots $5, 6, 7$.

   For example, to encode the message 1010, we would first place

   $$\_\_1\_010;$$

   then, because $1 + 0 + 0 = 1, 1 + 1 + 0 = 0, 0 + 1 + 0 = 1$, we would fill in the remaining slots to get

   $$1011010.$$

   This is a 2-ary code of length 7. Find its information density and its minimum distance.

4. Show that the maximum number of elements in a 2-ary code C of length 4 and distance 3 is 2.

5. Create a 4-ary code of length 4 and distance 3, that contains 16 elements.