Latin Squares	Instructor: Padraic Bartlett

Homework 7: Latin Squares and Chess

Week 3 — *Mathcamp 2012*

Attempt all of the problems that seem interesting, and let me know if you see any typos! (+) problems are harder than the others. (++) problems are currently open.

1. Construct a pandiagonal Latin square of order 7, and use it to solve the 7-queens problem.

2. While in class we said that pandiagonal Latin squares exist only when n is neither divisible by 2 or 3, it turns out that solutions to the n-queens problem exist for every value of $n \geq 4$. Using symmetry arguments, how many solutions can you find for $n = 5$? How about $n = 6$? (Hint: if you've done this problem correctly, there should be more solutions for $n = 5$ than $n = 6$. This is the only pair of numbers where this happens; in general, as n increases, the number of solutions to the n-queens problem grows exponentially, though it is an open question to how fast this exponential growth precisely is.)

3. Prove the claim we made in class, that there are no pandiagonal Latin squares with order divisible by 3. Do this via the following outline:

   (a) A **superdiagonal** of a $n \times n$ grid is a collection of n cells within this grid that contains exactly one representative from each row and column, as well as exacty one representative from each broken left diagonal and exactly one representative from each broken right diagonal.

   Show that a pandiagonal Latin square of order n exists if and only if it is possible to break the cells of a $n \times n$ grid up into n disjoint superdiagonals.

   (b) Show that a $n \times n$ array cannot have a superdiagonal if n is a multiple of 2 or 3.