Latin Squares	Instructor: Padraic Bartlett

Homework 5: Geometry and Latin Squares

Week 3 — *Mathcamp 2012*

Attempt all of the problems that seem interesting, and let me know if you see any typos! (+) problems are harder than the others. (++) problems are currently open.

1. Construct an affine plane of order 4.

2. A **projective plane** is a different geometrical notion, formed by the following three rules:

   (P1): Given any two points, there is a unique line joining any two points.

   (P2): Any two distinct lines intersect at a unique point.

   (P3): There are four points, no three of which are collinear. (This rule is just to eliminate the silly case where all of your points are on the same line.)

   Basically, we have removed our earlier axiom of ''Given any line L and point P, there is exactly one line parallel to L through P" with the axiom "There are no parallel lines."

   Find a projective plane containing 7 points.

3. Show that in a projective plane, there is a well-defined notion of "order," just like for affine planes: i.e. show that for any projective plane, there is some value of n such that

   - every line contains exactly $n + 1$ points,
   - every point is intersected by exactly $n + 1$ lines,
   - our plane contains $n^2 + n + 1$ points, and
   - our plane contains $n^2 + n + 1$ lines.

4. Find a projective plane of order 4.

5. Show that we can turn any affine plane of order n into a projective plane of order n, and vice-versa. (Hint: starting with an affine plane, split it into $n+1$ parallel classes. To each class C_i, add one point ∞_i, and extend every line in C_i to contain ∞_i. Then collect all of the ∞_i points into a line. What have you done? How could you undo this process?)