Latin Squares	Instructor: Padraic Bartlett

Homework 4: Groups and Latin Squares

Week 2 — *Mathcamp 2012*

Attempt all of the problems that seem interesting, and let me know if you see any typos! (+) problems are harder than the others. (++) problems are currently open.

1. For what values of n can you find a Latin square that does not come from a group table?

2. Prove the proposition I asked you to do in class:

   Proposition. If $A_1, \ldots A_n$ is a set of of mutually orthogonal row-Latin squares, then given any other row-Latin square X, the set $X \circ A_1, \ldots X \circ A_n$ is another set of mutually orthogonal Latin squares.

3. ($\frac{+}{2}$) Assume the following claim: Any group G can be written as a subgroup of S_n, for n equal to the number of elements in G. Use this to prove that a Latin square L is the multiplication table of a group if and only if the composition of any two rows in L is another row in L.

4. Using the finite field methods we described today, make 7 MOLS of order 8. (Don't explicitly write them out; rather, write out their general form, and write out two to test that they're actually orthogonal.)

5. Using the groups and graph theory methods we described today (if we finished them!), create 2 MOLS of order 15.

6. Which of the following Latin squares are multiplication tables of groups?

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 4 & 3 & 2 & 1 \\ 3 & 4 & 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 2 & 6 & 4 & 5 \\ 2 & 3 & 1 & 5 & 6 & 4 \\ 5 & 6 & 4 & 1 & 2 & 3 \\ 6 & 4 & 5 & 3 & 1 & 2 \\ 4 & 5 & 6 & 2 & 3 & 1 \end{bmatrix}.$$