Latin Squares	Instructor: Padraic Bartlett

Homework 1: Partial Latin Squares

Week 2	*Mathcamp 2012*

Attempt all of the problems that seem interesting, and let me know if you see any typos! (+) problems are harder than the others. (++) problems are currently open.

1. Construct a partial Latin square containing n filled cells that cannot be completed to a proper Latin square.

2. Do (1), but make sure that you don't use any symbol more than once.

3. Again, do (1), but make sure that you don't use any row more than once.

4. Find a completion of the following partial Latin square:

$$\begin{bmatrix} 1 & - & - & 2 \\ - & - & - & - \\ 3 & - & - & - \\ - & - & - & - \end{bmatrix}$$

   How many different completions of this partial Latin square can you find?

5. What is the smallest possible number of filled cells in a 4×4 partial Latin square P, so that there is exactly one way to complete P to a proper Latin square?

6. Show that if P is a 4×4 partial Latin square in which at most 3 cells are filled, P can be completed to a proper Latin square.

7. (+) Generalize (6) to $n \times n$ squares: i.e. show that if P is a $n \times n$ partial Latin square containing $\leq n-1$ filled cells, then P can be completed to a proper Latin square.

8. (++) Call a $n \times n$ partial Latin square $\frac{1}{4}$-dense if no row, column, or symbol is used more than $\frac{n}{4}$ many times. A question that I've studied for about the last year or so is the following: can any $\frac{1}{4}$-dense partial Latin square is completeable to a proper Latin square? (Let me know if you solve this! Or if you want to see work I've done on this problem.)

9. Show that this claim is the best we can hope for, in the following sense: find a 4×4 partial Latin square P in which no row, column, or symbol is used more than $\frac{4}{4}+1$ many times, that cannot be completed to a proper Latin square.

10. Generalize this: for any n, find a $n \times n$ partial Latin square P such that no rows, columns, or symbols are used more than $\frac{n}{4}+1$ many times, such that P cannot be completed to a proper Latin square.