The Unit Distance Graph | Instructor: Paddy

Homework 4: "Nice" Colorings

Week 4 | *Mathcamp 2010*

1. Find a way to fold up a hexagon to get a one-hole torus; similarly, find a way to fold up both a 10-gon and a 8-gon to get a two-hole torus. Can you generalize this?

2. Find a "nice" coloring of the torus that uses only 7 colors.

3. Is there a value of n such that if G is a connected, locally finite, locally Hamiltonian graph with $\geq n$ vertices, it must have a vertex of degree ≥ 7?

4. In the statement of Thomassen's 7-color-theorem, we said that if S was a surface and k was a natural number such that

   (a) every noncontractible simple closed curve has diameter ≥ 2,

   (b) every simple closed curve C with diameter < 2 is such that the area of $int(C)$ is $\leq k$, and

   (c) the diameter of S is $\geq 12k + 30$,

   then for any graph G planarly embedded on S, we need at least 7 colors to nicely color the faces of G.

   Show that each of the above enumerated conditions are necessary: i.e. for each pair $\{(a), (b)\}$,$\{(a), (c)\}$, $\{(b), (c)\}$, find a surface S and graph G that satisfies this pair of conditions and yet can be nicely colored with ≤ 6 colors.