

Homework 1: Model Theory!, the HW

Week 4

Mathcamp 2010

1. Write a sentence φ in the language $\mathcal{L} = \{R\}$, where R is a binary relation, that says that a structure $M \models \varphi$ iff R is an equivalence relation on M .
2. Write a sentence φ in the language $\mathcal{L} = \{R\}$, where R is a binary relation, so that the following holds: If M is a structure that models φ , then there must be a $n \in \mathbb{N}$ such that the size of M 's universe is $2n$.
3. Write a sentence φ in the language $\mathcal{L} = \{R, f\}$, where R is a binary relation and f is a binary function, so that the following holds: If M is a structure that models φ , then there must be a $n \in \mathbb{N}$ such that the size of M 's universe is n^2 .
4. Suppose that S is a structure for the language $\{<\}$ that satisfies the following three sentences:

$$\forall x \exists y (x < y)$$

$$\forall x \forall y \forall z ((x < y) \wedge (y < z) \Rightarrow (x < z))$$

$$\forall x \neg(x < x)$$

What can you say about the size of S ?

5. Let $S = \langle \mathbb{N}, < \rangle$ be the normal structure of the natural numbers under the ordering $<$, and let $T = \langle \mathbb{N}, <_T \rangle$ be the ordering on \mathbb{N} defined as follows:

$(n <_t m)$ holds iff $(n, m$ are both even or both odd, and $n < m)$ or $(n$ is even and m is odd.)

Find a sentence φ in the language $\{<\}$ such that S satisfies φ , but T does not.