Latin Squares	Instructor: Paddy

Homework 2: Applications of Latin Squares

Week 4 — *Mathcamp 2010*

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1. Using Hall's marriage theorem, deduce the following alternate form for it:

   Theorem 1 *Suppose that $G = (A, B)$ is a bipartite graph that satisfies*

   $$(\star): \qquad \forall H \subset A, |N(H)| \geq |H|, \text{ and } |A| = |B|.$$

   Then G has a 1-factor.

2. A **diagonal latin square** is a latin square in which both of the diagonals don't contain any repeated elements. Show that for n odd and not a multiple of 3, that there are diagonal $n \times n$ latin squares.

3. Show, furthermore, that there are a pair of mutually orthogonal diagonal latin squares of order n, if n is odd and not a multiple of 3.

4. A **magic square** is a $n \times n$ grid consisting of the integers $\{0, \ldots n^2 - 1\}$ such that the sum of any row, column, or diagonal is always the same value. Use the above result to prove that a $n \times n$ magic square exists whenever n is odd and not a multiple of 3.

5. Use the algorithm developed in class to complete the following partial latin square:

$$\begin{bmatrix}
1 & & & & & & & & & & \\
& 1 & & & & & & & & & \\
& & 2 & & & & & & & & \\
& & & 2 & & & & & & & \\
& & & & 3 & & & & & & \\
& & & & & 3 & & & & & \\
& & & & & & 4 & & & & \\
& & & & & & & 4 & & & \\
& & & & & & & & & & \\
& & & & & & & & & 5 & \\
& & & & & & & & & & 5
\end{bmatrix}$$