An Introduction to Graph Theory	Instructors: Marisa and Paddy

Homework 3: More trees!

Week 1 of 1 *Mathcamp 2010*

This problem set will continue to discuss some of the properties of trees. Problems here are (roughly, kinda) sorted from easy to difficult; use this judgement when you're deciding which questions to study.

1. Show that a graph is a tree if and only if it has a unique spanning tree.
2. If G is a graph, show that the largest acyclic subgraph of G is a forest found by taking spanning trees from each of G's connected components.
3. Show that any connected graph on n vertices with m edges has at least $m - n + 1$ cycles.
4. Show that any connected graph on n vertices has one cycle if *and only if* it has n edges. Is it true that such a graph has $m - n + 1$ cycles if and only if it has m edges?
5. A *graceful labeling* of a graph with E edges is a labeling $l(v)$ of its vertices with distinct integers from the set $\{0 \dots E\}$, such that each edge $\{u, v\}$ is uniquely determined by the difference $|l(u) - l(v)|$.

   Show that all path-trees[1] are graceful.
6. A *caterpillar tree* is a tree such that deleting all of its leaves leaves us with a single path (i.e. they kinda look like caterpillars.)

   Show that all caterpillar trees are graceful.[2]

[1] A path-tree is a tree that consists of a single path.

[2] The Ringel-Kotzig conjecture is the claim that all trees are graceful; graph theorists have been trying since about the 70's to prove this claim, to no avail. Let me know if you do!