Graph Colorings	Instructors: Marisa and Paddy

Homework 2: The Chromatic Polynomial

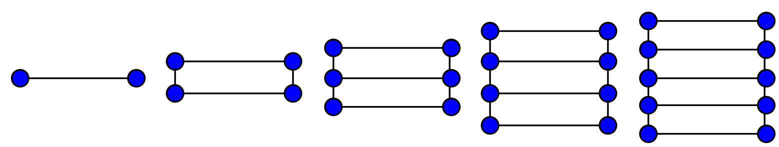
Week 2 — *Mathcamp 2010*

1. Prove that $P(C_n; t) = (t-1)^n + (-1)^n \cdot (t-1)$, where C_n is the cycle graph on n vertices.[1]

2. For a graph G with $E(G) \neq \emptyset$, prove that the sum of the coefficients of the chromatic polynomial of G is 0.

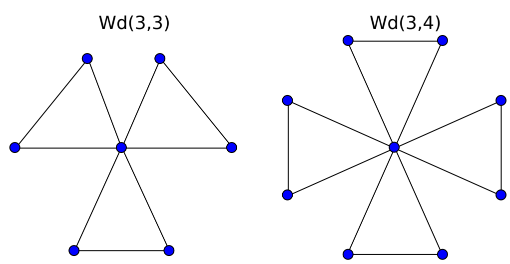
3. Prove that the coefficient on t^{n-1} in the chromatic polynomial of G is the number of edges in G.

4. The ladder graph L_n on $2n$ vertices is the graph formed by connecting two paths of length n as depicted below:



   Find the chromatic polynomial of L_n for as many n as you can (ideally, find a general formula for all n.)

5. The windmill graph $Wd(k, n)$ on $n(k-1)+1$ vertices is the graph formed by taking n copies of K_k and joining them all together at a common vertex, as shown below:



   Show that the chromatic polynomial of $Wd(n, k)$ is

$$\prod_{i=0}^{k-1} (x-i)^n.$$

[1] Hint: First, consider the case where $n = 3$ and compute $P(C_3; x)$. (Maybe compute $P(C_4; x)$, too, if you want practice.) Then proceed by induction; you will need the deletion/contraction theorem and the chromatic polynomial of a tree.