

Practice Final!

Week 10

Caltech 2012

1. Determine whether the following series converge:

(a)

$$\sum_{n=1}^{\infty} \frac{1}{(\ln(n))^k}.$$

(b)

$$\sum_{n=1}^{\infty} \frac{1}{(\ln(n))^n}.$$

(c)

$$\sum_{n=1}^{\infty} \frac{\sin\left(\frac{1}{n}\right)}{n}.$$

2. Evaluate the improper integral

$$\int_2^{\infty} \frac{1}{x\sqrt{x^2-1}} dx.$$

3. Use Taylor polynomials to approximate $\sin(.8)$ to within $\pm 10^{-4}$.

4. (a) Find the Taylor series for $\ln(1+x^6)$.

(b) Using the power series above, what complex power series would you use to define $f(z) = \ln(1+x^6)$ in the complex plane?

(c) What is the radius of convergence R of this power series?

(d) Find two values of $z \in \mathbb{C}$ with $\|z\| = R$ such that $f(z)$ converges, and two more values of $z \in \mathbb{C}$, $\|z\| = R$ such that $f(z)$ diverges.